# NEWTONIAN TORSION PHYSICS 

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#### Abstract

The purpose of this book is to explore concepts related to FREE Energy and the control of Gravity/Antigravity that are based entirely within the framework of extended classical Newtonian physics called Newtonian Torsion Physics. It is shown that the cause of gravity is a type of macroscopic torque between inertial frames where the origin of the torque occurs within imaginary or complex space. The effect manifests in real space as universal mass attraction, or gravity. A correlation has been discovered between mass, inductors, and capacitors, thereby relating the imaginary or complex origin of torque to gravity, and how this same torque affects electromagnetism. A simplified torsional mass relativity model called Natural Relativity (NR) Theory is presented and directly related to gravity. This theory is then correlated to Einstein's Special Relativity (SR) Theory, and as a consequence, creates a corrected "Principle of Equivalence Theorem" showing the origin of gravity and antigravity comes from imaginary or complex space. A temporal rotation operator is introduced using Euler's Identity, which shows the imaginary or complex (i.e., time-past and time-future) motion of matter as positive or negative displacement into imaginary space, which embodies the fundamental concept of time travel. The speed of light $c$, Planck's constant $h$, permeability $\mu_{0}$, permittivity $\varepsilon_{0}$, Boltzmann's Constant $k$, electric charge $q$, and the Fine Structure Constant $\alpha$ are invariant between inertial frames and therefore, unaffected by torque because the fluctuation or curvature of the fundamental parameters that compose these constants are shown to evaluate to unity gain. In other words, these constants remain constant anywhere within a given torque field or gravity well. Gravitomagnetic Theory shows that the magnetic field energy produced by a moving electron is shown to be a special relativistic mass fluctuation, and therefore creates a real torque, which couples to gravity as a secondary gravitational effect. This motion can either have a real velocity, or an imaginary or complex (i.e., time-past or time-future) velocity. If the velocity is complex, then the special relativistic mass fluctuation or $\pm M e$ of an electron can be either positive or negative, and exhibits a corresponding gravitational or antigravitational effect, which consequently produces a complex (i.e., time-past or timefuture) magnetic field. This is the origin of the effect called gravity or antigravity. In addition, the volume of the total field energy of a complex magnetic field can either be positive or negative, which can add or subtract from the energy produced by a real magnetic field. In the Bohr model of the Hydrogen atom, an Amperian Current is described as an electron circulating around a nucleus at a relativistic speed. This creates a magnetic induction emerging from the center of the nucleus. Fluctuating this field by applying an external magnetic induction causes the velocity of the electron to become complex. The presence of NEGATIVE RESISTANCE, the production of NEGATIVE ENERGY, and the control of GRAVITY/ANTIGRAVITY occur by fluctuating the mass of an object in complex space. The theory presents a conceptual breakthrough for the development of energy and high-speed field propulsion technologies.


## INTRODUCTION

Puthoff (1996) coined the phrase, "metric engineering", and Puthoff, Little and Ibison (2002) consider the vacuum to be a polarizable medium, and that it can be expressed in terms of tensor formulations of curved space-time. The bending of light passing near a massive object is caused by induced spatial variation in the refractive index of the vacuum near the object. This is correlated to changes in permeability $\mu_{0}$ and permittivity $\varepsilon_{0}$ of the vacuum. Changes occurring in the vacuum also affect the mass of objects, the length and bending of rulers, the frequency of clocks, the energy of light, etc. This book both simplifies and links gravity with electromagnetism by presenting formulations of curved space-time in terms of extended classical Newtonian physics, which are caused by relativistic fluctuations of mass $M$, inductance $L$, and capacitance $C$ of an object. For example, when an object
with mass $M$ naturally falls downward in a given gravity well, its' natural relativistic mass $+\dot{M}$ increases due to Newtonian Gravitation, or universal mass attraction. The new mass of an object $M+\dot{M}$ is displaced to a new position within this well, then mass-energy remains conserved by returning the mass to its place of origin, or $M+\dot{M}-\dot{M}=M$. In other words, by converting this increase in relativistic mass $+\dot{M}$ to energy $+\dot{E}$, a force acts upon the object, and the new mass is now displaced back to its original position that was higher vertically in the well. The object exhibits an antigravitational effect by removing or subtracting $\dot{M}$ from $M$. The rate of change of this fluctuation could cause the speed of the object to easily exceed the speed of light. This is because the relativistic gravitational mass of the object, which is shown to be convergent, is moving at right angles to a relativistic inertial mass, which is shown to be divergent. Since the speed of the object with relativistic gravitational mass has no known upper limit, the resulting speed through deep space could be enormous and necessitates the use of the warp factor equation.

## EVERYTHING IN THIS UNIVERSE IS CURVED!



FIGURE 1. Two identical spheres as seen by an observer on the Earth.
Mass and volume of matter vary under the influence of different gravity. Shown in Fig. 1 above are two identical spheres, SPHERE 1 and SPHERE 2, with equal mass and volume as seen by an observer on the Earth. SPHERE 1 is now moved to the Moon as shown in Fig. 2 below.


FIGURE 2. Two identical spheres change in mass and volume relative to position of the observer.

SPHERE 1 on the left is under the influence of gravity of the Moon and SPHERE 2 on the right remains under the influence of gravity of the Earth. Since the gravity of the Moon $g_{\text {moon }}$ is approximately $1 / 6$ the gravity of the Earth $g_{0}$, an observer on the Moon would measure the sphere on the Earth as having slightly more mass $+\dot{M}$ and being slightly smaller in volume $-\dot{v}$ than the same sphere on the Moon. Likewise, an observer on the Earth would measure the sphere on the Moon as having slightly less mass $-\dot{M}$, and slightly larger in volume $+\dot{v}$. This is due to the effect of curvature of space and time upon matter caused by torsion originating from imaginary or complex space and observed in real space as universal mass attraction, or gravity.

## APPLYING THE PRODUCT RULE

Marmet (2001) considers "separately" the influence of a gravitational potential upon matter, and assumes for the moment that kinetic energy is zero. The kinetic energy and gravitational energy are evaluated independently and therefore are considered mutually exclusive. For Kinetic Energy systems, and given a frame of reference, the following inertial-based parameters mass $M$, inductor $L$, and capacitor $C$, are considered invariant. However, for Gravitational Energy systems, and given an equipotential surface of gravity reference, the following torsionalbased parameters relativistic mass $\pm \Delta M$, relativistic inductor $\pm \Delta L$, and relativistic capacitor $\pm \Delta C$, fluctuate or curve between inertial frames. So, by applying the product rule, this principle is mathematically expressed as,

$$
n(t)=\frac{d}{d t}(a b)=b \frac{d a}{d t}+a \frac{d b}{d t}=b \dot{a}+a \dot{b} \approx b\left(\frac{\Delta a}{\Delta t}\right)+a\left(\frac{\Delta b}{\Delta t}\right)
$$

Where, the Inertial Term is given as,

$$
b \dot{a}=0
$$

And the Torsional Term is given as,

$$
a \dot{b} \neq 0
$$

So,

$$
n(t)=a \dot{b}=a \frac{d b}{d t} \approx a\left(\frac{\Delta b}{\Delta t}\right)
$$

The first term is regarded as inertial, and therefore is considered kinetic or fluxes, and is Newtonian-based. The second term is regarded as torsional, and therefore, causes gravity. When evaluating Gravitational Energy systems, the Inertial Term must be equal to zero, leaving only the Torsional Term to be determined. The Torsional Term may result in a positive or negative number meaning that the change in Gravitational Energy may be either positive or negative. This principle extends Newtonian-based physics to include Newtonian Torsion Physics, and is formulated and analyzed in the direction of gravity, or towards a center of mass. A positive change in Gravitational Energy indicates displacement towards a center of mass and a negative change in Gravitational Energy indicates displacement away from a center of mass.

## TORSIONAL MASS FLUCTUATION



FIGURE 3. The fluctuating mass of an object due to gravity.
Applying the product rule, the complete ideal momentum model is composed of two terms,

$$
\begin{equation*}
f_{M}(t)=\frac{d p_{Y}}{d t}=\frac{d\left(M v_{Y}\right)}{d t}=M \frac{d v_{Y}}{d t}+v_{Y} \frac{d M}{d t}=M \dot{v}_{Y}+v_{Y} \dot{M} \tag{1}
\end{equation*}
$$

Where, the Inertial Term is $M \dot{v}_{Y}$, and mass $M$ is invariant within any equipotential surface of gravity $g_{Y}$. The Torsional Term is $v_{Y} \dot{M}$, and changing mass $\dot{M}$ fluctuates between equipotential surfaces of gravity.

For a "mass fluctuating system", the Torsional Term is NOT zero Newtons. So, given an object having mass $M$ moving at constant velocity $v_{Y}$, or $\dot{v}_{Y}=0 \mathrm{~m} / \mathrm{s}^{2}$, the Inertial Term is,

$$
\begin{equation*}
f_{M}(t)=M \dot{v}_{Y}=0 N \tag{2}
\end{equation*}
$$

This removes the Inertial Term, leaving only the Torsional Term,

$$
\begin{equation*}
f_{M}(t)=v_{Y} \dot{M} \neq 0 N \tag{3}
\end{equation*}
$$

Since $\dot{M}$ has units of resistance in $m N s / m^{2}$, its direction of change could either be POSITIVE or NEGATIVE. If $\dot{M}$ is negative, it has units of negative resistance or,

$$
\begin{equation*}
\dot{M}<0 \mathrm{mNs} / \mathrm{m}^{2} \tag{4}
\end{equation*}
$$

Now, the instantaneous gravitationally induced power $P_{M}$ of a fluctuating mass $\dot{M}$ is,

$$
\begin{equation*}
P_{M}(t)=v_{Y} f_{M}(t)=v_{Y}^{2} \dot{M} \tag{5}
\end{equation*}
$$

So, for certain values of $\dot{M}$, the total instantaneous power $P_{M}$ can be NEGATIVE or,

$$
\begin{equation*}
P_{M}(t)<0 \text { Watts } \tag{6}
\end{equation*}
$$

Then, integrating $P_{M}$ with respect to time when the total power is less than zero watts results in NEGATIVE energy of mass $M$ or,

$$
\begin{equation*}
E_{M}(t)=\int P_{M}(t) d t=v_{Y}^{2} \int \dot{M} d t=v_{Y}^{2} M(t)<0 \text { Joules } \tag{7}
\end{equation*}
$$

If $\dot{M}$ is positive, it has units of positive resistance or,

$$
\begin{equation*}
\dot{M}>0 \mathrm{mNs} / \mathrm{m}^{2} \tag{8}
\end{equation*}
$$

Now, the instantaneous gravitationally induced power $P_{M}$ of a fluctuating mass $\dot{M}$ is,

$$
\begin{equation*}
P_{M}(t)=v_{Y} f_{M}(t)=v_{Y}^{2} \dot{M} \tag{9}
\end{equation*}
$$

So, for certain values of $\dot{M}$, the total instantaneous power $P_{M}$ can be POSITIVE or,

$$
\begin{equation*}
P_{M}(t)>0 \text { Watts } \tag{10}
\end{equation*}
$$

Then, integrating $P_{M}$ with respect to time results in excess POSITIVE energy of mass $M$ or,

$$
\begin{equation*}
E_{M}(t)=\int P_{M}(t) d t=v_{Y}^{2} \int \dot{M} d t=v_{Y}^{2} M(t)>0 \text { Joules } \tag{11}
\end{equation*}
$$

So, the energy equivalent of mass (gravitational energy) $E_{M}$ is,

$$
\begin{equation*}
E_{M}(t)=M(t) v_{Y}^{2} \tag{12}
\end{equation*}
$$

By rearranging terms, the mass equivalent of energy $M$ is,

$$
\begin{equation*}
M(t)=\frac{E_{M}(t)}{v_{Y}{ }^{2}} \tag{13}
\end{equation*}
$$

The fluctuating mass equivalent of energy $\dot{M}$ is,

$$
\begin{equation*}
\dot{M}=\frac{\dot{E}_{M}}{v_{Y}^{2}} \tag{14}
\end{equation*}
$$

So, the gravitational momentum model is,

$$
\begin{equation*}
f_{M}(t)=\dot{M} v_{Y}=\frac{\dot{E}_{M}}{v_{Y}{ }^{2}} v_{Y}=\frac{\dot{E}_{M}}{v_{Y}} \tag{15}
\end{equation*}
$$

Letting $\dot{y}=d y / d t=v_{Y}$, the Torsional Force Term is,

$$
\begin{equation*}
f_{M}(t)=\frac{\dot{E}_{M}}{\dot{y}} \tag{16}
\end{equation*}
$$

By rearranging terms, the fluctuating energy equivalent of mass $\dot{E}_{M}$ is,

$$
\begin{equation*}
\dot{E}_{M}=f_{M}(t) \dot{y} \tag{17}
\end{equation*}
$$

The total energy $E_{M}$ contained within matter is,

$$
\begin{equation*}
E_{M}(t)=M(t) c^{2} \tag{18}
\end{equation*}
$$

The change in this total energy $\dot{E}_{M}$ is,

$$
\begin{equation*}
\dot{E}_{M}=\dot{M} c^{2} \tag{19}
\end{equation*}
$$

By rearranging terms, the change in total mass $\dot{M}$ is,

$$
\begin{equation*}
\dot{M}=\frac{\dot{E}_{M}}{c^{2}} \tag{20}
\end{equation*}
$$

So, the time derivative form of the Torsional Mass Term is,

$$
\begin{equation*}
\dot{M}=\frac{f_{M}(t) \dot{y}}{c^{2}} \tag{21}
\end{equation*}
$$

The derivative form is,

$$
\begin{equation*}
d M=\frac{f_{M} d y}{c^{2}} \tag{22}
\end{equation*}
$$

The difference form is,

$$
\begin{equation*}
\Delta M=\frac{f_{M} \Delta y}{c^{2}} \tag{23}
\end{equation*}
$$

## TORSIONAL INDUCTIVE MASS FLUCTUATION



FIGURE 4. The fluctuating inductance of an object due to gravity.
Applying the product rule, the complete ideal inductor model is composed of two terms,

$$
\begin{equation*}
v_{L}(t)=\frac{d}{d t}\left(L I_{L}\right)=L \frac{d I_{L}}{d t}+I_{L} \frac{d L}{d t}=L \dot{I}_{L}+I \dot{L}_{L} \tag{24}
\end{equation*}
$$

Where, the Inertial Term is $L \dot{I}_{L}$, and inductance $L$ is invariant within any equipotential surface of gravity $g_{Y}$. The Torsional Term is $I \dot{L}_{L}$, changing inductance $\dot{L}$ fluctuates between equipotential surfaces of gravity.

For an "inductive fluctuating system", the Torsional Term is NOT zero volts. By applying a constant current $I_{L}$ through inductor $L$, or $\dot{I}_{L}=0 \mathrm{Amps} / \mathrm{s}$, the Inertial Term is,

$$
\begin{equation*}
v_{L}(t)=L \dot{I}_{L}=0 \text { Volts } \tag{25}
\end{equation*}
$$

This removes the Inertial Term, leaving only the Torsional Term,

$$
\begin{equation*}
v_{L}(t)=I_{L} \dot{L} \neq 0 \text { Volts } \tag{26}
\end{equation*}
$$

Since $\dot{L}$ has units of resistance in ohms, $\Omega$, its direction of change could either be POSITIVE or NEGATIVE. If $\dot{L}$ is negative, it has units of negative resistance or,

$$
\begin{equation*}
\dot{L}<0 \Omega \tag{27}
\end{equation*}
$$

Now, the instantaneous gravitationally induced power $P_{L}$ of a fluctuating inductor $\dot{L}$ is,

$$
\begin{equation*}
P_{L}(t)=I_{L} v_{L}(t)=I_{L}^{2} \dot{L} \tag{28}
\end{equation*}
$$

So, for certain values of $\dot{L}$, the total instantaneous power $P_{L}$ can be NEGATIVE or,

$$
\begin{equation*}
P_{L}(t)<0 \text { Watts } \tag{29}
\end{equation*}
$$

Then, integrating $P_{L}$ with respect to time when the total power is less than zero watts results in NEGATIVE energy of inductor $L$ or,

$$
\begin{equation*}
E_{L}(t)=\int P_{L} d t=I_{L}^{2} \int \dot{L} d t=I_{L}^{2} L(t)<0 \text { Joules } \tag{30}
\end{equation*}
$$

If $\dot{L}$ is positive, it has units of positive resistance or,

$$
\begin{equation*}
\dot{L}>0 \Omega \tag{31}
\end{equation*}
$$

Now, the instantaneous gravitationally induced power $P_{L}$ of a fluctuating inductor $\dot{L}$ is,

$$
\begin{equation*}
P_{L}(t)=I_{L} v_{L}(t)=I_{L}^{2} \dot{L} \tag{32}
\end{equation*}
$$

So, for certain values of $\dot{L}$, the total instantaneous power $P_{L}$ can be POSITIVE or,

$$
\begin{equation*}
P_{L}(t)>0 \text { Watts } \tag{33}
\end{equation*}
$$

Then, integrating $P_{L}$ with respect to time results in excess POSITIVE energy of inductor $L$ or,

$$
\begin{equation*}
E_{L}(t)=\int P_{L} d t=I_{L}^{2} \int \dot{L} d t=I_{L}^{2} L(t)>0 \text { Joules } \tag{34}
\end{equation*}
$$

Equate this to the energy equivalent of mass $E_{M}$,

$$
\begin{equation*}
E_{L}(t)=E_{M}(t) \tag{35}
\end{equation*}
$$

Then, the energy equivalent of mass (gravitational energy) $E_{L}$ is,

$$
\begin{equation*}
E_{L}(t)=v_{Y}^{2} M_{L}(t)=I_{L}^{2} L(t) \tag{36}
\end{equation*}
$$

By rearranging terms, the mass equivalent of energy $M_{L}$ is,

$$
\begin{gather*}
M_{L}(t)=\frac{E_{L}(t)}{v_{Y}^{2}}=\frac{I_{L}^{2} L(t)}{v_{Y}^{2}}  \tag{37}\\
v_{Y}^{2}=\frac{I_{L}^{2} L(t)}{M_{L}(t)} \tag{38}
\end{gather*}
$$

The fluctuating mass equivalent of energy $\dot{M}_{L}$ is,

$$
\begin{equation*}
\dot{M}_{L}=\frac{\dot{E}_{L}}{v_{Y}{ }^{2}}=\frac{I_{L}{ }^{2} \dot{L}}{v_{Y}{ }^{2}} \tag{39}
\end{equation*}
$$

So, the gravitational inductor model is,

$$
\begin{equation*}
f_{L}(t)=\dot{M}_{L} v_{Y}=\frac{\dot{E}_{L}}{v_{Y}{ }^{2}} v_{Y}=\frac{\dot{E}_{L}}{v_{Y}}=\frac{I_{L}^{2} \dot{L}}{v_{Y}} \tag{40}
\end{equation*}
$$

Letting $\dot{y}=d y / d t=v_{Y}$, the Torsional Inductive Force Term is,

$$
\begin{equation*}
f_{L}(t)=\frac{\dot{E}_{L}(t)}{\dot{y}} \tag{41}
\end{equation*}
$$

By rearranging terms, the fluctuating energy equivalent of mass $\dot{E}_{L}$ is,

$$
\begin{equation*}
\dot{E}_{L}=f_{L}(t) \dot{y} \tag{42}
\end{equation*}
$$

The total energy $E_{L}$ contained within matter is,

$$
\begin{equation*}
E_{L}(t)=M_{L}(t) c^{2} \tag{43}
\end{equation*}
$$

The change in this total energy $\dot{E}_{L}$ is,

$$
\begin{equation*}
\dot{E}_{L}=\dot{M}_{L} c^{2} \tag{44}
\end{equation*}
$$

By rearranging terms, the change in total mass $\dot{M}$ is,

$$
\begin{equation*}
\dot{M}_{L}=\frac{\dot{E}_{L}}{c^{2}}=\frac{f_{L}(t) \dot{y}}{c^{2}}=\frac{I_{L}^{2} \dot{L}}{\dot{y}^{2}}=\frac{I_{L}^{2} \dot{L}}{\frac{I_{L}^{2} L(t)}{M_{L}(t)}}=\frac{M_{L}(t) \dot{L}}{L(t)} \tag{45}
\end{equation*}
$$

So, the time derivative form of the Torsional Inductive Term is,

$$
\begin{equation*}
\dot{L}=\frac{f_{L}(t) L(t) \dot{y}}{M_{L}(t) c^{2}} \tag{46}
\end{equation*}
$$

The derivative form is,

$$
\begin{equation*}
d L=\frac{f_{L} L d y}{M_{L} c^{2}} \tag{47}
\end{equation*}
$$

The difference form is,

$$
\begin{equation*}
\Delta L=\frac{f_{L} L \Delta y}{M_{L} c^{2}} \tag{48}
\end{equation*}
$$

## TORSIONAL CAPACITIVE MASS FLUCTUATION



FIGURE 5. The fluctuating capacitance of an object due to gravity.
Applying the product rule, the complete ideal capacitor model is composed of two terms,

$$
\begin{equation*}
i_{C}(t)=\frac{d}{d t}\left(C V_{C}\right)=C \frac{d V_{C}}{d t}+V_{C} \frac{d C}{d t}=C \dot{V}_{C}+V_{C} \dot{C} \tag{49}
\end{equation*}
$$

Where, the Inertial Term is $C \dot{V}_{C}$, and capacitor $C$ is invariant within any equipotential surface of gravity $g_{Y}$. The Torsional Term is $V_{C} \dot{C}$, and changing capacitance $\dot{C}$ fluctuates between equipotential surfaces of gravity.

For a "capacitive fluctuating system", the Torsional Term is NOT zero amps. By applying a constant voltage across capacitor $C$, or $\dot{V}_{C}=0$ Volts $/ s$, the Inertial Term is,

$$
\begin{equation*}
i_{C}(t)=C \dot{V}_{C}=0 \mathrm{Amps} \tag{50}
\end{equation*}
$$

This removes Inertial Term, leaving only the Torsional Term,

$$
\begin{equation*}
i_{C}(t)=V_{C} \dot{C} \neq 0 \mathrm{Amps} \tag{51}
\end{equation*}
$$

Since $\dot{C}$ has units of conductance in mhos, $\mho$, its direction of change could either be POSITIVE or NEGATIVE. If $\dot{C}$ is negative, it has units of negative conductance or,

$$
\begin{equation*}
\dot{C}<0 \mho \tag{52}
\end{equation*}
$$

Now, the instantaneous gravitationally induced power $P_{C}$ of a fluctuating capacitor $\dot{C}$ is

$$
\begin{equation*}
P_{C}(t)=i_{C}(t) V_{C}=V_{C}^{2} \dot{C} \tag{53}
\end{equation*}
$$

So, for certain values of $\dot{C}$, the total instantaneous power $P_{C}$ can be NEGATIVE or,

$$
\begin{equation*}
P_{C}(t)<0 \text { Watts } \tag{54}
\end{equation*}
$$

Then, integrating $P_{C}$ with respect to time when the total power is less than zero watts results in NEGATIVE energy of capacitor $C$ or,

$$
\begin{equation*}
E_{C}(t)=\int P_{C} d t=V_{C}^{2} \int \dot{C} d t=V_{C}^{2} C(t)<0 \text { Joules } \tag{55}
\end{equation*}
$$

If $\dot{C}$ is positive, it has units of positive conductance or,

$$
\begin{equation*}
\dot{C}>0 \mho \tag{56}
\end{equation*}
$$

Now, the instantaneous gravitationally induced power $P_{C}$ of a fluctuating capacitor $\dot{C}$ is,

$$
\begin{equation*}
P_{C}(t)=i_{C}(t) V_{C}=V_{C}^{2} \dot{C} \tag{57}
\end{equation*}
$$

So, for certain values of $\dot{C}$, the total instantaneous power $P_{C}$ can be POSITIVE or

$$
\begin{equation*}
P_{C}(t)>0 \text { Watts } \tag{58}
\end{equation*}
$$

Then, integrating $P_{C}$ with respect to time results in excess POSITIVE energy of capacitor $C$ or,

$$
\begin{equation*}
E_{C}(t)=\int P_{C} d t=V_{C}^{2} \int \dot{C} d t=V_{C}^{2} C(t)>0 \text { Joules } \tag{59}
\end{equation*}
$$

Equate this to the energy equivalent of mass $E_{M}$,

$$
\begin{equation*}
E_{C}(t)=E_{M}(t) \tag{60}
\end{equation*}
$$

Then, the energy equivalent of mass (gravitational energy) $E_{C}$ is,

$$
\begin{equation*}
E_{C}(t)=v_{Y}^{2} M_{C}(t)=V_{C}^{2} C(t) \tag{61}
\end{equation*}
$$

By rearranging terms, the mass equivalent of energy $M_{C}$ is,

$$
\begin{gather*}
M_{C}(t)=\frac{E_{C}(t)}{v_{Y}^{2}}=\frac{C(t) V_{C}^{2}}{v_{Y}^{2}}  \tag{62}\\
v_{Y}^{2}=\frac{V_{C}^{2} C(t)}{M_{C}(t)} \tag{63}
\end{gather*}
$$

The fluctuating mass equivalent of energy $\dot{M}_{C}$ is,

$$
\begin{equation*}
\dot{M}_{C}=\frac{\dot{E}_{C}}{v_{Y}{ }^{2}}=\frac{V_{C}{ }^{2} \dot{C}}{v_{Y}{ }^{2}} \tag{64}
\end{equation*}
$$

So, the gravitational capacitor model is,

$$
\begin{equation*}
f_{C}(t)=\dot{M}_{C} v_{Y}=\frac{\dot{E}_{C}}{v_{Y}^{2}} v_{Y}=\frac{\dot{E}_{C}}{v_{Y}}=\frac{V_{C}^{2} \dot{C}}{v_{Y}} \tag{65}
\end{equation*}
$$

Letting $\dot{y}=d y / d t=v_{Y}$, the Torsional Capacitive Force Term is,

$$
\begin{equation*}
f_{C}(t)=\frac{\dot{E}_{C}}{\dot{y}} \tag{66}
\end{equation*}
$$

By rearranging terms, the fluctuating energy equivalent of mass $\dot{E}_{C}$ is,

$$
\begin{equation*}
\dot{E}_{C}=f_{L}(t) \dot{y} \tag{67}
\end{equation*}
$$

The total energy $E_{C}$ contained within matter is,

$$
\begin{equation*}
E_{C}(t)=M_{C}(t) c^{2} \tag{68}
\end{equation*}
$$

The change in this total energy $\dot{E}_{C}$ is,

$$
\begin{equation*}
\dot{E}_{C}=\dot{M}_{C} c^{2} \tag{69}
\end{equation*}
$$

By rearranging terms, the change in total mass $\dot{M}$ is,

$$
\begin{equation*}
\dot{M}_{C}=\frac{\dot{E}_{C}}{c^{2}}=\frac{f_{C}(t) \dot{y}}{c^{2}}=\frac{V_{C}^{2} \dot{C}}{\dot{y}^{2}}=\frac{V_{C}^{2} \dot{C}}{\frac{V_{C}^{2} C(t)}{M_{C}(t)}}=\frac{M_{C}(t) \dot{C}}{C(t)} \tag{70}
\end{equation*}
$$

So, the time derivative form of the Torsional Capacitive Term is,

$$
\begin{equation*}
\dot{C}=\frac{f_{C}(t) C(t) \dot{y}}{M_{C}(t) c^{2}} \tag{71}
\end{equation*}
$$

The derivative form is,

$$
\begin{equation*}
d C=\frac{f_{C} C d y}{M_{C} c^{2}} \tag{72}
\end{equation*}
$$

The difference form is,

$$
\begin{equation*}
\Delta C=\frac{f_{C} C \Delta y}{M_{C} c^{2}} \tag{73}
\end{equation*}
$$

## THE GRAVITATIONAL COUPLING OF A FLUCTUATING MASS, INDUCTOR OR CAPACITOR



> ACCELERATION DUE TO GRAVITY
> $\mathrm{m} / \mathrm{sec}^{2}$

FIGURE 6. The natural mass fluctuation of an object due to gravitational free fall.
Natural universal mass attraction or classic Newtonian gravity is a force $f_{y}$ that acts through a center of mass of the Earth with mass $M_{E}$ and a test mass $M$ separated by a distance $y$,

$$
\begin{equation*}
f_{y}=\frac{G M_{E} M}{y^{2}}=g_{y} M \tag{74}
\end{equation*}
$$

So, given,
Gravitational constant $G=6.67259 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$
Radius of Earth $y_{E}=6.3781 \times 10^{6} \mathrm{~m}$

The surface gravity $g_{0}$ of the Earth is,

$$
\begin{equation*}
g_{0}=\frac{G M_{E}}{y_{E}{ }^{2}}=\frac{\left(6.67259 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)^{2}}=9.80665 \mathrm{~m} / \mathrm{sec}^{2} \tag{75}
\end{equation*}
$$

The gravity $g_{y_{0}}$ at GRAVITATIONAL REFERENCE position $y_{0}$ is,

$$
\begin{equation*}
g_{y_{0}}=\frac{G M_{E}}{y_{0}{ }^{2}}=\frac{\left(6.67259 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(19.920 \times 10^{6} \mathrm{~m}\right)^{2}}=1.00537 \mathrm{~m} / \mathrm{sec}^{2} \tag{76}
\end{equation*}
$$

The gravity $g_{y_{1}}$ at position $y_{1}$ is,

$$
\begin{equation*}
g_{y_{1}}=\frac{G M_{E}}{y_{1}{ }^{2}}=\frac{\left(6.67259 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(7.529 \times 10^{6} \mathrm{~m}\right)^{2}}=7.03767 \mathrm{~m} / \mathrm{sec}^{2} \tag{77}
\end{equation*}
$$

Marmet (2001) equates force $f_{M}$ produced by a fluctuating test mass $d M$ to the gravitational force $f_{y}$,

$$
\begin{equation*}
f_{y}(t)=f_{M}(t) \tag{78}
\end{equation*}
$$

The derivative form of a fluctuating test mass $d M$ is,

$$
\begin{equation*}
d M=\frac{f_{M} d y}{c^{2}}=\frac{g_{y} M d y}{c^{2}} \tag{79}
\end{equation*}
$$

The derivative form of the gravitational RED SHIFT (or BLUE SHIFT) of test mass $d M / M$ and the energy equivalent of test mass $d E / E$ displaced $d y$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{d M}{M}=\frac{d E_{M}}{E_{M}}=\frac{g_{y} d y}{c^{2}} \tag{80}
\end{equation*}
$$

Marmet states the equation above shows a "calculated change of energy levels as a function of gravitational potential is in perfect agreement with the Pound and Rebka and also the Pound and Snider experiments". Therefore, energy increases as a function of downward or positive displacement within a given gravity well.

Let $d Y_{g_{y}}=g_{y} d y$, the exponential solutions of the derivative form of mass and energy equivalent of mass are,

$$
\begin{gather*}
\int_{M_{y_{0}}}^{M_{y_{1}}} \frac{1}{M} d M=\frac{1}{c^{2}} \int_{g_{y_{0}} y_{0}}^{g_{y_{1}} y_{1}} d Y_{g_{y}}  \tag{81}\\
\left.\ln (M)\right|_{M_{y_{0}}} ^{M_{y_{1}}}=\left.\frac{1}{c^{2}} Y_{g_{y}}\right|_{g_{y_{0}} y_{0}} ^{g_{y_{1}} y_{1}}  \tag{82}\\
\ln \left(M_{y_{1}}\right)-\ln \left(M_{y_{0}}\right)=\ln \left(\frac{M_{y_{1}}}{M_{y_{0}}}\right)=\frac{1}{c^{2}}\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)  \tag{83}\\
\left.\frac{M_{y_{1}}}{M_{y_{0}}}=e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right.}\right)  \tag{84}\\
M_{y_{1}}=M_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{85}
\end{gather*}
$$

$$
\begin{equation*}
E_{M y_{1}}=E_{M y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0} y_{0}}}{c^{2}}\right)} \tag{86}
\end{equation*}
$$

So, the Pound, Rebka and Snider experiment used Mossbauer spectroscopy to measure the electromagnetic gravitational RED SHIFT (or BLUE SHIFT) of 14.4 keV gamma rays emitted from $\mathrm{Fe}{ }^{57}$ through a vertical distance of 22.6 m . With the gamma rays emitted upward, they showed the RED SHIFT was within one percent (1\%) of this result,

$$
\begin{gather*}
\text { SHIFT }=\frac{\Delta M}{M}=\frac{\Delta E_{M}}{E_{M}}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}=\frac{G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}}  \tag{87}\\
\text { SHIFT }=\frac{\left(6.67259 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)\left(\frac{1}{\left(6.3781226 \times 10^{6} \mathrm{~m}\right)}-\frac{1}{\left(6.3781000 \times 10^{6} \mathrm{~m}\right)}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}} \tag{88}
\end{gather*}
$$

$$
\begin{equation*}
\text { SHIFT }=-2.465961 \times 10^{-15} \tag{89}
\end{equation*}
$$

With the gamma rays emitted downward, they showed the BLUE SHIFT was,

$$
\begin{equation*}
\text { SHIFT }=\frac{\left(6.67259 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)\left(\frac{1}{\left(6.3781000 \times 10^{6} \mathrm{~m}\right)}-\frac{1}{\left(6.3781226 \times 10^{6} \mathrm{~m}\right)}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}} \tag{90}
\end{equation*}
$$

$$
\begin{equation*}
\text { SHIFT }=2.465961 \times 10^{-15} \tag{91}
\end{equation*}
$$

Now, the force $f_{L}$ produced by a fluctuating inductor $d L$ is equated to the gravitational force $f_{y}$,

$$
\begin{equation*}
f_{y}(t)=f_{L}(t) \tag{92}
\end{equation*}
$$

The derivative form of a fluctuating inductor $d L$ is,

$$
\begin{equation*}
d L=\frac{f_{L} L d y}{M_{L} c^{2}}=\frac{g_{y} L d y}{c^{2}} \tag{93}
\end{equation*}
$$

The derivative form of the gravitational RED SHIFT (or BLUE SHIFT) of inductor $d L / L$ displaced $d y$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{d L}{L}=\frac{g_{y} d y}{c^{2}} \tag{94}
\end{equation*}
$$

Let $d Y_{g_{y}}=g_{y} d y$, the exponential solution of the derivative form of an inductor is,

$$
\begin{equation*}
\int_{L_{y_{0}}}^{L_{y_{1}}} \frac{1}{L} d L=\frac{1}{c^{2}} \int_{g_{y_{0}} y_{0}}^{g_{y_{1}} y_{1}} d Y_{g_{y}} \tag{95}
\end{equation*}
$$

$$
\begin{gather*}
\left.\ln (L)\right|_{L_{y_{0}}} ^{L_{y_{1}}}=\left.\frac{1}{c^{2}} Y_{g_{y}}\right|_{g_{y_{0}} y_{0}} ^{g_{y_{1}} y_{1}}  \tag{96}\\
\ln \left(L_{y_{1}}\right)-\ln \left(L_{y_{0}}\right)=\ln \left(\frac{L_{y_{1}}}{L_{y_{0}}}\right)=\frac{1}{c^{2}}\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)  \tag{97}\\
\frac{L_{y_{1}}}{L_{y_{0}}}=e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0} y_{0}}}{c^{2}}\right)}  \tag{98}\\
L_{y_{1}}=L_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{99}
\end{gather*}
$$

So, the gravitational RED SHIFT (or BLUE SHIFT) of an inductor $L$ is,

$$
\begin{equation*}
\text { SHIFT }=\frac{\Delta L}{L}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}=\frac{G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}} \tag{100}
\end{equation*}
$$

Now, the force $f_{C}$ produced by a fluctuating capacitor $d C$ is equated to the gravitational force $f_{y}$,

$$
\begin{equation*}
f_{y}(t)=f_{C}(t) \tag{101}
\end{equation*}
$$

The derivative form of a fluctuating inductor $d C$ is,

$$
\begin{equation*}
d C=\frac{f_{C} C d y}{M_{C} c^{2}}=\frac{g_{y} C d y}{c^{2}} \tag{102}
\end{equation*}
$$

The derivative form of the gravitational RED SHIFT (or BLUE SHIFT) of capacitor $d C / C$ displaced $d y$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{d C}{C}=\frac{d E_{C}}{E_{C}}=\frac{g_{y} d y}{c^{2}} \tag{103}
\end{equation*}
$$

Let $d Y_{g_{y}}=g_{y} d y$, the exponential solution of the derivative form of a capacitor is,

$$
\begin{gather*}
\int_{C_{y_{0}}}^{C_{y_{1}}} \frac{1}{C} d C=\frac{1}{c^{2}} \int_{g_{y_{0}}}^{g_{y_{1}} y_{1}} d Y_{g_{y}}  \tag{104}\\
\left.\ln (C)\right|_{C_{y_{0}}} ^{C_{y_{1}}}=\left.\frac{1}{c^{2}} Y_{g_{y}}\right|_{g_{y_{0} y_{0}}} ^{g_{y_{1} y_{1}}}  \tag{105}\\
\ln \left(C_{y_{1}}\right)-\ln \left(C_{y_{0}}\right)=\ln \left(\frac{C_{y_{1}}}{C_{y_{0}}}\right)=\frac{1}{c^{2}}\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right) \tag{106}
\end{gather*}
$$

$$
\begin{gather*}
\frac{C_{y_{1}}}{C_{y_{0}}}=e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{107}\\
C_{y_{1}}=C_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0} y_{0}}}{c^{2}}\right)} \tag{108}
\end{gather*}
$$

So, the gravitational RED SHIFT (or BLUE SHIFT) of a capacitor $C$ is,

$$
\begin{equation*}
\text { SHIFT }=\frac{\Delta C}{C}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}=\frac{G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}} \tag{109}
\end{equation*}
$$

## NATURAL RELATIVITY THEORY



FIGURE 7. A change of relativistic mass due to gravity.
The establishment of a GRAVITATIONAL REFERENCE is defined as a fixed plane of reference within a given gravity well $g_{y}$. This plane is described as an equipotential surface of gravity, and is used throughout this book. Natural relativistic changes of mass, volume, frequency, energy, etc. fluctuate or curve as a function of displacement $\pm \Delta y$ from this plane of reference. This displacement defines a new plane of equipotential surface of gravity, and the gravity at that point may be increased or decreased based upon the sign of the displacement. Marmet (2001) invokes the principle of mass-energy conservation regarding the displacement of matter between planes. For example, an object of mass $M$ displaced a distance $\Delta y$ changes back to its original mass when returned to its original position within the same gravity well. Therefore, a new and simplified relativity model is introduced. For gravitational energy systems, and given an equipotential surface of gravity reference, the following parameters including relativistic mass $\pm \Delta M$, relativistic inductance $\pm \Delta L$, and relativistic capacitance $\pm \Delta C$, fluctuate or curve between equipotential surfaces of gravity by displacement $\pm \Delta y$. Again, the kinetic energy of gravitational energy systems is assumed to be zero. So, given a common equipotential surface of gravity reference $g_{y}$, an increase in gravity causes a natural relativistic increase in mass, energy equivalent of mass (gravitational energy), inductance, and capacitance. Likewise, a decrease in gravity causes a natural relativistic decrease in the same metrics.

Given an object with a rest mass $M_{y_{0}}$, an equivalent energy of the rest mass $E_{M y_{0}}$, an inductance $L_{y_{0}}$, and a capacitance $C_{y_{0}}$, the new rest mass $M_{y_{1}}$, the new equivalent energy of the rest mass $E_{M y_{1}}$, the new inductance $L_{y_{1}}$, and the new capacitance $C_{y_{1}}$ are,

$$
\begin{gather*}
M_{y_{1}}=\gamma_{N R} M_{y_{0}}=M_{y_{0}} \pm \Delta M_{y_{0}}  \tag{110}\\
E_{M y_{1}}=\gamma_{N R} E_{M y_{0}}=E_{M y_{0}} \pm \Delta E_{M y 0}  \tag{111}\\
L_{y_{1}}=\gamma_{N R} L_{y_{0}}=L_{y_{0}} \pm \Delta L_{y_{0}}  \tag{112}\\
C_{y_{1}}=\gamma_{N R} C_{y_{0}}=C_{y_{0}} \pm \Delta C_{y_{0}} \tag{113}
\end{gather*}
$$

The natural relativistic gamma $\gamma_{N R}$ is,

$$
\begin{equation*}
\gamma_{N R}=e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{114}
\end{equation*}
$$

The difference forms and the exponential forms of the natural relativistic mass $M_{y_{1}}$ model, energy equivalent of mass $E_{M y_{1}}$ model, inductor $L_{y_{1}}$ model, and capacitor $C_{y_{1}}$ model at position $\pm \Delta y$ within a given gravity well $g_{y}$ are,

$$
\begin{align*}
& M_{y_{1}}=M_{y_{0}} \pm \Delta M_{y_{0}}=M_{y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=M_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{115}\\
& E_{M y_{1}}=E_{M y_{0}}+\Delta E_{M y_{0}}=E_{M y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=E_{M y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{116}\\
& L_{y_{1}}=L_{y_{0}}+\Delta L_{y_{0}}=L_{y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=L_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{117}\\
& C_{y_{1}}=C_{y_{0}} \pm \Delta C_{y_{0}}=C_{y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=C_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{118}
\end{align*}
$$

In summary, Natural Relativity (NR) Theory describes the nature of the primary gravitational effect.

## SPECIAL RELATIVITY THEORY



FIGURE 8. The definition of a space-time interval relative to a stationary Observer A.
Einstein (1905) formulated his theory of special relativity and is described as Observer $B$, moving at velocity $v_{x}$ relative to a stationary Observer $A$, undergoes a relativistic effect. This effect changes various properties of Observer $B$ such as mass, lengths, time intervals, frequencies, and energy with respect to stationary Observer $A$. It's expressed as a Pythagorean-type quantity called a space-time interval, and evaluates as a Lorentz temporal correction shown below.

$$
\begin{gather*}
d s^{2}=d s_{0}^{2}+d x^{2} \\
d s_{0}^{2}=d s^{2}-d x^{2}(120) \\
d s_{0}^{2}=c^{2} d t^{2}-d x^{2} \tag{121}
\end{gather*}
$$

Since the velocity $v_{x}$ of Observer $B$ is,

$$
\begin{equation*}
v_{x}=\dot{x}=\frac{d x}{d t} \tag{122}
\end{equation*}
$$

$$
\begin{equation*}
d x=v_{x} d t \tag{123}
\end{equation*}
$$

So,

$$
\begin{equation*}
d s_{0}^{2}=c^{2} d t^{2}-v_{x}^{2} d t^{2} \tag{124}
\end{equation*}
$$

Then,

$$
\begin{equation*}
d s_{0}=\sqrt{c^{2} d t^{2}-\frac{c^{2} d t^{2} v_{x}^{2} d t^{2}}{c^{2} d t^{2}}}=c d t \sqrt{1-\frac{v_{x}^{2}}{c^{2}}} \tag{125}
\end{equation*}
$$

Since the speed of light $c$ is,

$$
\begin{gather*}
c=\dot{s}_{0}=\frac{d s_{0}}{d t}  \tag{126}\\
d s_{0}=c d t_{0} \tag{127}
\end{gather*}
$$

So, the Lorentz temporal correction relative to Observer $B$ is,

$$
\begin{gather*}
c d t_{0}=c d t \sqrt{1-\frac{v_{x}^{2}}{c^{2}}}  \tag{128}\\
d t_{0}=d t \sqrt{1-\frac{v_{x}^{2}}{c^{2}}}  \tag{129}\\
d t=\frac{d t_{0}}{\sqrt{1-\frac{v_{x}^{2}}{c^{2}}}} \tag{130}
\end{gather*}
$$

Therefore, relative to stationary Observer $A$ with a time interval $d t_{A}$, Observer $B$ 's clock at time interval $d t_{B}$ will be ticking slower or RED SHIFTED, and have the following Lorentz temporal correction,

$$
\begin{equation*}
d t_{B}=d t_{A} \sqrt{1-\frac{v_{B}^{2}}{c^{2}}} \tag{131}
\end{equation*}
$$

For $v_{B}>0$,

$$
\begin{align*}
& d t_{A}>d t_{B}  \tag{132}\\
& \Delta t_{A}>\Delta t_{B}
\end{align*}
$$

Likewise, relative to Observer $B$ in motion with a time interval $d t_{B}$, stationary Observer $A$ 's clock with a time interval $d t_{A}$ will be ticking faster or BLUE SHIFTED, and have the following Lorentz temporal correction,

$$
\begin{equation*}
d t_{A}=\frac{d t_{B}}{\sqrt{1-\frac{v_{B}^{2}}{c^{2}}}} \tag{133}
\end{equation*}
$$

For $v_{B}>0$,

$$
\begin{align*}
& d t_{B}<d t_{A}  \tag{134}\\
& \Delta t_{B}<\Delta t_{A}
\end{align*}
$$



FIGURE 9. A change of relativistic mass due to imaginary or complex velocity.
Given the rest mass of an object $M$, the special relativistic mass $M_{v}$ model presented by Einstein (1905) shows an object moving at velocity $v_{x}$ is,

$$
\begin{equation*}
M_{v}=\frac{M}{\sqrt{1-\frac{v_{x}{ }^{2}}{c^{2}}}}=\gamma M=M+d M \tag{135}
\end{equation*}
$$

The gamma $\gamma$ is,

$$
\begin{equation*}
\gamma=\frac{1}{\sqrt{1-\frac{v_{x}^{2}}{c^{2}}}} \tag{136}
\end{equation*}
$$

Implementing the binomial expansion of the above equation,

$$
\begin{equation*}
\gamma_{S R}=1+\frac{1}{2} \frac{v_{x}^{2}}{c^{2}}+\frac{3}{8} \frac{v_{x}^{4}}{c^{4}}+\frac{5}{16} \frac{v_{x}^{6}}{c^{6}}+\frac{35}{128} \frac{v_{x}^{8}}{c^{8}}+\ldots \tag{137}
\end{equation*}
$$

Using the 1 st order term of the expansion shown above where $v_{x} \ll c$, the new special relativistic gamma $\gamma_{S R}$ is,

$$
\begin{equation*}
\gamma_{S R}=1+\frac{v_{x}^{2}}{2 c^{2}} \tag{138}
\end{equation*}
$$

The derivative form of the special relativistic mass $M_{v}$ model moving at velocity $v_{x}$ is,

$$
\begin{equation*}
M_{v}=\gamma_{S R} M=M \pm d M=M\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right) \tag{139}
\end{equation*}
$$

The derivative form of a fluctuating mass $d M$ is,

$$
\begin{equation*}
d M=\frac{M v_{\chi}{ }^{2}}{2 c^{2}} \tag{140}
\end{equation*}
$$

An object can move at a real (i.e., time-forward) velocity $v_{x}$, at an imaginary (i.e., time-future) velocity $j v_{x}$, or at a velocity that is a combination of the two. The real and imaginary components are rotated about the temporal axis and therefore, can be described as complex motion. The rotation is given as $0^{\circ} \leq \theta \leq 90^{\circ}$, where the real axis is $\theta=0^{\circ}$ and the imaginary time-future axis is $\theta=90^{\circ}$. The complex number uses the Euler's identity $e^{j \theta}$, which functions as a temporal rotation operator.

$$
\begin{equation*}
v_{x}=v e^{j \theta}=v \cos \theta+j v \sin \theta \tag{141}
\end{equation*}
$$

So, the derivative form of the inertial RED SHIFT (or BLUE SHIFT) of mass $d M / M$, energy equivalent of mass $d E_{M} / E_{M}$, inductor $d L / L$, or capacitor $d C / C$ of an object moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
\text { SHIFT }=\frac{d M}{M}=\frac{d E_{M}}{E_{M}}=\frac{d L}{L}=\frac{d C}{C}=\frac{v_{x}^{2}}{2 c^{2}} \tag{142}
\end{equation*}
$$

The exponential solution of the derivative form of mass is,

$$
\begin{gather*}
\int_{M_{0}}^{M_{v}} \frac{1}{M} d M=\frac{v_{x}^{2}}{2 c^{2}}  \tag{143}\\
\left.\ln (M)\right|_{M_{0}} ^{M_{v}}=\frac{v_{x}^{2}}{2 c^{2}}  \tag{144}\\
\ln \left(M_{v}\right)-\ln \left(M_{0}\right)=\ln \left(\frac{M_{v}}{M_{0}}\right)=\frac{v_{x}^{2}}{2 c^{2}}  \tag{145}\\
\frac{M_{v}}{M_{0}}=e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{146}\\
M_{v}=M_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{147}\\
E_{M_{v}}=E_{M 0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{148}\\
L_{v}=L_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{149}\\
C_{v}=C_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{150}
\end{gather*}
$$

So, the inertial RED SHIFT (or BLUE SHIFT) of a mass $M$, an energy equivalent of mass $E_{M}$, an inductor $L$, and a capacitor $C$ is,

$$
\begin{equation*}
\text { SHIFT }=\frac{\Delta M}{M}=\frac{\Delta E_{M}}{E_{M}}=\frac{\Delta L}{L}=\frac{\Delta C}{C}=\frac{v_{x}{ }^{2}}{2 c^{2}} \tag{151}
\end{equation*}
$$

The special relativistic gamma $\gamma_{S R}$ is,

$$
\begin{equation*}
\gamma_{S R}=e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{152}
\end{equation*}
$$

The difference forms and the exponential forms of the special relativistic mass $M_{v}$ model, energy equivalent of mass $E_{M v}$ model, inductor $L_{v}$ model, and capacitor $C_{v}$ model of an object moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ are,

$$
\begin{gather*}
M_{v}=M_{0} \pm \Delta M_{0}=M_{0}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=M_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{153}\\
E_{M v}=E_{M 0} \pm \Delta E_{M 0}=E_{M 0}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=E_{M 0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{154}\\
L_{v}=L_{0} \pm \Delta L_{0}=L_{0}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=L_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{155}\\
C_{v}=C_{0}+\Delta C_{0}=C_{0}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=C_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{156}
\end{gather*}
$$

## DID EINSTEIN GET IT RIGHT, OR NOT?

Einstein's General Relativity Theory (1916) equates Newton's second law of motion, $f=m_{i} a$, where $m_{i}$ is the inertial mass to Newton's gravitational force, $f=m_{g} g$, where $m_{g}$ is the gravitational mass. Is this concept correct? Einstein used this to formulate his equivalence principle and stated, "There is no experiment a person could conduct in a small volume of space that would distinguish between a gravitational field and an equivalent uniform acceleration". Is this statement correct? Lets test Einstein's principle in the following thought experiment:


FIGURE 10. An elevator at rest on the Earth is NOT equivalent to an elevator accelerating in space.
As shown above, the natural relativistic mass fluctuation of an elevator at rest on the surface of the Earth is zero, or $d M=0$. However, the second order special relativistic mass fluctuation of the same elevator accelerating in space is non-zero or $d^{2} M>0$, and as a consequence, radiates electromagnetic waves. According to Woodward (1998), radiation reaction is observed in bodies being accelerated based upon Newton's second law of motion, $f_{0}=m a$. Therefore, given this scenario, the gravitational mass can't be equivalent to its' inertial mass due to their differences in mass fluctuations.


FIGURE 11. An elevator in free fall above the Earth is equivalent to an elevator moving at constant velocity in space.
As shown above, an elevator traveling a distance $+d y$ in free fall and the same elevator moving at a constant velocity $v_{x}$ at right angles to free fall produce virtually no radiation reaction. So, given this second scenario, these mass fluctuations are considered equivalent, hence, establishing a new Principle of Equivalence Theorem.

A NEW PRINCIPLE OF EQUIVALENCE THEOREM


FIGURE 12. Velocity profile of a 1 kg inertial mass.

The inertial RED SHIFT of an object due to velocity $v_{x}$ is,

$$
\begin{equation*}
\text { SHIFT }=\frac{\Delta M}{M}=\frac{v_{x}{ }^{2}}{2 c^{2}} \tag{157}
\end{equation*}
$$



FIGURE 13. Displacement profile of a 1 kg gravitational mass due to Earth's gravity well.
And since the gravitational RED SHIFT of the same object due to gravity $g_{y}$ is,

$$
\begin{equation*}
\text { SHIFT }=\frac{\Delta M}{M}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}=\frac{G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}} \tag{158}
\end{equation*}
$$

By equating an inertial RED SHIFT to a gravitational RED SHIFT, a new Principle of Equivalence Theorem is determined as,

$$
\begin{equation*}
\frac{v_{x}^{2}}{2 c^{2}}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}=\frac{G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}} \tag{159}
\end{equation*}
$$

So, an object displaced $\pm \Delta y$ within the Earth’s gravity well $g_{y}$ is equivalent to the same object moving at complex velocity $v_{x}$,

$$
\begin{equation*}
v_{x}=\sqrt{2\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}=\sqrt{2 G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{160}
\end{equation*}
$$



FIGURE 14. Equating an inertial RED (BLUE) SHIFT to a gravitational RED (BLUE) SHIFT.
As shown above, if the displacement $\Delta y=y_{0}-y_{1}$ of an object is POSITIVE, then the object is moving at a real velocity $v_{x}$. However, if the displacement $\Delta y$ is NEGATIVE, then the same object is moving at a complex (i.e., time-future) velocity $\sqrt{-1} v_{x}$, or velocity $j v_{x}$, where $j=\sqrt{-1}$. The real and imaginary components are rotated
about the temporal axis as a complex velocity. The rotation is given as $0^{\circ} \leq \theta \leq 90^{\circ}$, where the real axis is $\theta=0^{\circ}$ and the imaginary time-future axis is $\theta=90^{\circ}$. The complex number uses the Euler's identity $e^{j \theta}$, which functions as a temporal rotation operator.

$$
\begin{equation*}
v_{x}=v e^{j \theta}=v \cos \theta+j v \sin \theta \tag{161}
\end{equation*}
$$

So, given an object moving at a complex velocity $v_{x}$, the equivalent displacement to position $y_{1}$ within the Earth's gravity well $g_{y}$, where $0<y_{1} \leq \infty$ or $-1 \leq \frac{y_{0} v_{x}{ }^{2}}{2 G M_{E}}$ is,

$$
\begin{equation*}
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{v_{x}^{2}}{2}\right)=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}} \tag{162}
\end{equation*}
$$

Therefore, the equivalent displacement $\Delta y$ within the Earth's gravity well $g_{y}$ is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=y_{0}-\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=y_{0}\left(1-\frac{1}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}\right) \tag{163}
\end{equation*}
$$

The equivalent maximum complex velocity $v_{x \max }$ at $y_{1}=\infty$ is

$$
\begin{gather*}
1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}=0  \tag{164}\\
v_{x \max }=\sqrt{-\frac{2 G M_{E}}{y_{0}}} \tag{165}
\end{gather*}
$$

So, given the equivalent maximum complex velocity $v_{x \max }$, the minimum mass $M_{\min }$ at $y_{1}=\infty$ is,

$$
\begin{equation*}
M_{\min }=M_{y_{0}}\left(1+\frac{v_{x}^{2}}{2 c^{2}}\right)=M_{y_{0}}\left(1-\frac{G M_{E}}{y_{0} c^{2}}\right)=M_{y_{0}} e^{\left(-\frac{G M_{E}}{y_{0} c^{2}}\right)} \tag{166}
\end{equation*}
$$

In summary, this new Principle of Equivalence Theorem describes an object moving at one-half the square of a real velocity $v_{x}$ is equivalent to the same object having fallen down a displacement $+d y$ within a given gravity well $g_{y}$. This object naturally acquires more relativistic mass, inductance and capacitance as it moves at a real velocity $v_{x}$, and augments its’ own gravitation with other objects. On the other hand, the same object moving at one-half the square of a complex velocity $j v_{x}$ is equivalent to the same object having fallen $u p$ a displacement $-d y$ in the same gravity well. This object naturally loses more relativistic mass, inductance and capacitance as it moves at a complex velocity $j v_{x}$, and diminishes its' own gravitation with other objects. Therefore, special relativity is considered to be a secondary gravitational effect.

Example 1. Given the velocity profile above of an object having a mass $M_{0}$ moving at a special relativistic timeforward velocity $v_{x}$, compute the new special relativistic inertial mass $M_{v}$.

So, given,
Direction of time $\theta=0^{\circ}$
Mass of object $M_{0}=1.0 \mathrm{~kg}$
Velocity of object $v=1.0 \times 10^{8} \mathrm{~m} / \mathrm{sec}$

The time-forward velocity $v_{x}$ of an object is,

$$
\begin{equation*}
v_{x}=v e^{j \theta}=\left(1.0 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right) e^{j 0^{\circ}}=1.0 \times 10^{8} \mathrm{~m} / \mathrm{sec} \tag{167}
\end{equation*}
$$

The new special relativistic inertial mass $M_{v}$ is,

$$
\begin{gather*}
M_{v}=M_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}=(1.0 \mathrm{~kg}) e^{\left(\frac{\left(1.0 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99793 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}\right)}  \tag{168}\\
M_{v}=1.05721 \mathrm{~kg} \tag{169}
\end{gather*}
$$

The inertial mass of the object was increased by,

$$
\begin{equation*}
M_{v}-M_{0}=(1.05721 \mathrm{~kg})-(1.0 \mathrm{~kg})=0.05721 \mathrm{~kg} \tag{170}
\end{equation*}
$$

Example 2. Given the velocity profile above of an object having a mass $M_{0}$ moving at a special relativistic velocity time-future $j v_{x}$, compute the new special relativistic inertial mass $M_{v}$.

So, given,
Direction of time $\theta=90^{\circ}$
Mass of object $M_{0}=1.0 \mathrm{~kg}$
Velocity of object $v=1.0 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
The time-future velocity $v_{x}$ of an object is,

$$
\begin{equation*}
v_{x}=v e^{j \theta}=\left(1.0 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right) e^{j 90^{\circ}}=1.0 j \times 10^{8} \mathrm{~m} / \mathrm{sec} \tag{171}
\end{equation*}
$$

The new special relativistic inertial mass $M_{v}$ is,

$$
\begin{align*}
M_{v}=M_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} & =(1.0 \mathrm{~kg}) e^{\left(\frac{\left(1.0 j \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99793 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}\right)}  \tag{172}\\
M_{v}= & 0.94589 \mathrm{~kg} \tag{173}
\end{align*}
$$

The inertial mass of the object was reduce by,

$$
\begin{equation*}
M_{v}-M_{0}=(0.94589 \mathrm{~kg})-(1.0 \mathrm{~kg})=-0.05411 \mathrm{~kg} \tag{174}
\end{equation*}
$$

Example 3. Given the gravitational profile above of an object having a mass $M_{y_{0}}$ displaced to a position $-\Delta y$ within Earth's gravity well $g_{y}$, compute the new natural relativistic gravitational mass $M_{y_{1}}$.

So, given,
Mass of object $M_{y_{0}}=1.0 \mathrm{~kg}$
Object on Earth's surface $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Object displaced to $y_{1}=1.0 \times 10^{8} \mathrm{~m}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The acceleration due to gravity at surface of Earth is,

$$
\begin{equation*}
g_{y_{0}}=\frac{G M_{E}}{y_{0}{ }^{2}}=\frac{\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)^{2}}=9.80665 \mathrm{~m} / \mathrm{sec}^{2} \tag{175}
\end{equation*}
$$

The acceleration due to gravity at altitude $y_{1}=1.0 \times 10^{8} \mathrm{~m}$ above the Earth is,

$$
\begin{equation*}
g_{y_{1}}=\frac{G M_{E}}{y_{1}^{2}}=\frac{\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(1.0 \times 10^{8} \mathrm{~m}\right)^{2}}=0.039894 \mathrm{~m} / \mathrm{sec}^{2} \tag{176}
\end{equation*}
$$

Given the exponential solution of the natural relativistic mass model, the new gravitational mass is,

$$
\begin{gather*}
M_{y_{1}}=M_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}=(1.0 \mathrm{~kg}) e^{\left(\frac{\left(0.039894 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(1.0 \times 10^{8} \mathrm{~m}\right)-\left(9.80665 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}\right)}  \tag{177}\\
M_{y_{1}}=0.9999999993485 \mathrm{~kg} \tag{178}
\end{gather*}
$$

The gravitational mass of the object was reduced by,

$$
\begin{equation*}
M_{y_{1}}-M_{y_{0}}=(0.9999999993485 \mathrm{~kg})-(1.0 \mathrm{~kg})=-6.516 \times 10^{-10} \mathrm{~kg} \tag{179}
\end{equation*}
$$

Example 4. Assuming there are no other gravitational influences besides the Earth, compute the new minimum natural relativistic gravitational mass $M_{\text {min }}$ of an object at $y_{1}=\infty$.

So, given,
Mass of object $M_{y_{0}}=1.0 \mathrm{~kg}$
Object on Earth's surface $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The minimum gravitational mass $M_{\text {min }}$ at $y_{1}=\infty$ is,

$$
\begin{gather*}
M_{\min }=M_{y_{0}} e^{\left(-\frac{G M_{E}}{y_{0} c^{2}}\right)}=(1.0 \mathrm{~kg}) e^{\left(-\frac{\left(6.67260 \times 10^{-11} N \mathrm{~m}^{2} / \mathrm{kg}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}\right)}  \tag{180}\\
M_{\min }=0.999999993041 \mathrm{~kg} \tag{181}
\end{gather*}
$$

The gravitational mass of the object was reduced by,

$$
\begin{equation*}
M_{y_{1}}-M_{y_{0}}=(0.9999999993041 \mathrm{~kg})-(1.0 \mathrm{~kg})=-6.959 \times 10^{-10} \mathrm{~kg} \tag{182}
\end{equation*}
$$

## HOW GRAVITY AFFECTS THE VOLUME OF OBJECTS

An object of volume $v_{y}$ (length $\mathscr{L}_{y}$, width $W_{y}$, and height $\mathcal{H}_{y}$ ) contracts as a function of position $+\Delta y$ within gravity well $g_{y}$. Likewise, the same the volume $v_{y}$ dilates as a function of position $-\Delta y$ in the same gravity well.

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of an object of volume $\Delta v / v$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta L}{L}=\frac{\Delta w}{w}=\frac{\Delta \nexists}{\nexists}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{183}
\end{equation*}
$$

The difference forms and exponential forms of the natural relativistic object of volume $\gamma_{y}$ at position $\pm \Delta y$ is,

$$
\begin{align*}
& \mathcal{L}_{y_{1}}=\mathcal{L}_{y_{0}} \mp \Delta \mathcal{L}_{y_{0}}=\mathcal{L}_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=\mathcal{L}_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{184}\\
& w_{y_{1}}=w_{y_{0}} \mp \Delta w_{y_{0}}=w_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=w_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{185}\\
& \mathcal{H}_{y_{1}}=\mathcal{H}_{y_{0}} \mp \Delta \mathcal{H}_{y_{0}}=\mathcal{H}_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=\mathcal{H}_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{186}
\end{align*}
$$

## HOW VELOCITY AFFECTS THE VOLUME OF OBJECTS

An object of volume $\boldsymbol{v}_{v}$ (length $\mathcal{L}_{v}$, width $w_{v}$, and height $\mathcal{H}_{v}$ ) contracts moving at a real velocity $v_{x}$. Likewise, the same object of volume $v_{v}$ dilates moving at a complex velocity $j v_{x}$.

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of an object of volume $\Delta v / v$ moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta L}{L}=\frac{\Delta w}{w}=\frac{\Delta \notin}{\nexists}=-\frac{v_{x}^{2}}{2 c^{2}} \tag{187}
\end{equation*}
$$

The difference forms and exponential forms of the special relativistic object of volume $v_{v}$ moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{align*}
& \mathscr{L}_{v}=\mathscr{L}_{0} \mp \Delta \mathscr{L}_{0}=\mathscr{L}_{0}\left(1 \mp \frac{v_{x}^{2}}{2 c^{2}}\right)=\mathscr{L}_{0} e^{\left(-\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{188}\\
& \boldsymbol{w}_{v}=\mathscr{w}_{0} \mp \Delta \mathscr{w}_{0}=\mathscr{w}_{0}\left(1 \mp \frac{v_{x}^{2}}{2 c^{2}}\right)=\mathscr{w}_{0} e^{\left(-\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{189}\\
& \mathscr{H}_{v}=\mathscr{H}_{0} \mp \Delta \mathscr{H}_{0}=\mathscr{H}_{0}\left(1 \mp \frac{v_{x}^{2}}{2 c^{2}}\right)=\mathscr{H}_{0} e^{\left(-\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{190}
\end{align*}
$$

## HOW GRAVITY AFFECTS THE FREQUENCY OF TIME

A mechanical oscillator vibrating at a frequency $f_{y}$ contracts (i.e., slows down) as a function of position $+\Delta y$ within gravity well $g_{y}$. Likewise, the same mechanical oscillator vibrating at a frequency $f_{y}$ dilates (i.e., speeds up) as a function of position $-\Delta y$ in the same gravity well.

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of an oscillator vibrating at a frequency $\Delta f / f$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta f}{f}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{191}
\end{equation*}
$$

The difference form and exponential form of the natural relativistic frequency $f_{y}$ of an oscillator at position $\pm \Delta y$ is,

$$
\begin{equation*}
f_{y_{1}}=f_{y_{0}} \mp \Delta f_{y_{0}}=f_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=f_{y_{0}} e^{\left(-\frac{g_{y_{1} y_{1}-g_{y_{0}} y_{0}}^{c^{2}}}{c^{2}}\right)} \tag{192}
\end{equation*}
$$

## HOW VELOCITY AFFECTS THE FREQUENCY OF TIME

A mechanical oscillator vibrating at a frequency $f_{v}$ contracts (i.e., slows down) while moving at a real velocity $v_{x}$. Likewise, the same mechanical oscillator vibrating at a frequency $f_{v}$ dilates (i.e., speeds up) while moving at a complex velocity $j v_{x}$.

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of an oscillator vibrating at a frequency $\Delta f / f$ while moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta f}{f}=-\frac{v_{x}{ }^{2}}{2 c^{2}} \tag{193}
\end{equation*}
$$

The difference form and exponential form of the special relativistic frequency $f_{v}$ of an oscillator moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
f_{v}=f_{0} \mp \Delta f_{0}=f_{0}\left(1 \mp \frac{v_{x}^{2}}{2 c^{2}}\right)=f_{0} e^{\left(-\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{194}
\end{equation*}
$$

## HOW GRAVITY AFFECTS AN INTERVAL OF TIME

A mechanical oscillator vibrating for an interval of time $t_{y}$ contracts (i.e., slows down) as a function of position $+\Delta y$ within gravity well $g_{y}$. Likewise, the same mechanical oscillator vibrating for an interval of time $t_{y}$ dilates (i.e., speeds up) as a function of position $-\Delta y$ in the same gravity well.

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of an oscillator vibrating for an interval of time $\Delta t / t$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta t}{t}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{195}
\end{equation*}
$$

The difference form and exponential form of the natural relativistic time interval $t_{y}$ of an oscillator at position $\pm \Delta y$ is,

$$
\begin{equation*}
t_{y_{1}}=t_{y_{0}} \mp \Delta t_{y_{0}}=t_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=t_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{196}
\end{equation*}
$$

## HOW VELOCITY AFFECTS AN INTERVAL OF TIME

A mechanical oscillator vibrating for an interval of time $t_{v}$ contracts (i.e., slows down) while moving at a real velocity $v_{x}$. Likewise, the same mechanical oscillator vibrating for an interval of time $t_{v}$ dilates (i.e., speeds up) while moving at a complex velocity $j v_{x}$.

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of an oscillator vibrating for an interval of time $\Delta t / t$ while moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta t}{t}=-\frac{v_{x}^{2}}{2 c^{2}} \tag{197}
\end{equation*}
$$

The difference form and exponential form of the special relativistic time interval $t_{v}$ of an oscillator moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
t_{v}=t_{0} \mp \Delta t_{0}=t_{0}\left(1 \mp \frac{v_{x}^{2}}{2 c^{2}}\right)=t_{0} e^{\left(-\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{198}
\end{equation*}
$$

## HOW GRAVITY AFFECTS LINEAR MOMENTUM

The momentum $p_{y}$ of an object of mass $M_{y}$ moving at velocity $v_{x}$ increases as a function of position $+\Delta y$ within gravity well $g_{y}$. Likewise, the momentum $p_{y}$ of the same object decreases as a function of position $-\Delta y$ in the same gravity well,

$$
\begin{equation*}
p_{y}=M_{y} v_{x} \tag{199}
\end{equation*}
$$

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of momentum $\Delta p / p$ of an object displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta p}{p}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{200}
\end{equation*}
$$

The difference form and exponential form of the natural relativistic momentum $p_{y}$ at position $\pm \Delta y$ is,

$$
\begin{equation*}
p_{y_{1}}=p_{y_{0}} \pm \Delta p_{y_{0}}=p_{y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=p_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{201}
\end{equation*}
$$

## HOW VELOCITY AFFECTS LINEAR MOMENTUM

The momentum $p_{v}$ of an object of mass $M_{0}$ increases moving at a real velocity $v_{x}$. Likewise, the momentum $p_{v}$ of the same object decreases moving at a complex velocity $j v_{x}$,

$$
\begin{equation*}
p_{v}=M_{0} v_{x} \tag{202}
\end{equation*}
$$

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of momentum $\Delta p / p$ of an object moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta p}{p}=\frac{v_{x}^{2}}{2 c^{2}} \tag{203}
\end{equation*}
$$

The difference form and exponential form of the special relativistic momentum $p_{v}$ of an object moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
p_{v}=p_{0} \pm \Delta p_{0}=p_{0}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=p_{0} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{204}
\end{equation*}
$$

## HOW GRAVITY AFFECTS ANGULAR MOMENTUM

The angular momentum $S_{y}$ of an object of mass $M_{y}$ moving at velocity $v_{x}$ with a radius $r$ is invariant as a function of position $+\Delta y$ within gravity well $g_{y}$. Likewise, the angular momentum $S_{y}$ of the same object is invariant as a function of position $-\Delta y$ in the same gravity well,

$$
\begin{equation*}
S_{y}=M_{y} v_{x} r \tag{205}
\end{equation*}
$$

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of angular momentum $\Delta S / S$ of an object displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta S}{S}=0 \tag{206}
\end{equation*}
$$

The difference form and exponential form of the natural relativistic angular momentum $S_{y}$ at position $\pm \Delta y$ is,

$$
\begin{equation*}
S_{y_{1}}=S_{y_{0}} \tag{207}
\end{equation*}
$$

## HOW VELOCITY AFFECTS ANGULAR MOMENTUM

The angular momentum $S_{v}$ of an object of mass $M_{0}$ is invariant moving at a real velocity $v_{x}$ with a radius $r$. Likewise, the angular momentum $S_{v}$ of the same object is invariant moving at a complex velocity $j v_{x}$,

$$
\begin{equation*}
S_{v}=M_{0} v_{x} r \tag{208}
\end{equation*}
$$

So, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of angular momentum $\Delta S / S$ of an object moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta S}{S}=0 \tag{209}
\end{equation*}
$$

The difference form and exponential form of the special relativistic angular momentum $S_{v}$ of an object moving at a real velocity $v_{x}$ or a complex velocity $j v_{x}$ is,

$$
\begin{equation*}
S_{v}=S_{0} \tag{210}
\end{equation*}
$$

## THE SPACE-TIME MEDIA OR THE AETHER



FIGURE 14. The space-time media or aether.
According to Puthoff (1996) and Puthoff, Little and Ibison (2002), the vacuum is described as having magnetic permeability $\mu_{0}$ and dielectric permittivity $\varepsilon_{0}$, and acts to impede the propagation of light and the motion of matter. Direct modification of these components changes the nature of light and matter.


FIGURE 15. Two similar objects undergoing natural universal mass attraction.
The "active vacuum" of space, or space-time media (i.e., the aether) is composed of uncondensed relativistic mass. An object $M_{1}$ made of matter (i.e., atoms) and given a GRAVITATIONAL REFERENCE point undergoes universal mass attraction (i.e., gravitational free fall) with another object $M_{2}$. Both objects acquire relativistic mass by a natural means from the surrounding space-time media as a function of displacement $+\Delta y$ between the two objects. This media condenses onto both objects as more relativistic mass, thereby increasing their total mass $M_{n}+\Delta M_{n}$, inductance $L_{n}+\Delta L_{n}$, and capacitance $C_{n}+\Delta C_{n}$. This action changes the relativistic momentum of both objects resulting with increasing force of attraction. The space-time media between these objects rarefy or relativistic mass condenses out of the media thereby affecting both the magnetic permeability $\mu_{0}$ and the dielectric permittivity $\varepsilon_{0}$ of free space. This rarefaction of media is referred to as a gravity well, and as a consequence, causes the volume of space occupied by both objects and the space between them to be reduced. The space-time media in a rarefying state means gravity between these objects is increasing, which causes light passing near these objects to amplify in energy $\varepsilon_{\lambda y}$ and increase in frequency $f_{\lambda y}$ as proven by the Pound and Rebka experiment (1964). This behavior of space-time media acts as an impedance upon the natural motion of matter and the propagation of light.

## THE GRAVITATIONAL COUPLING OF AN ELECTROMAGNETIC WAVE



FIGURE 16. Electromagnetic waves propagating within a given gravity well.
Shown above is the BLUE SHIFTING of an electromagnetic wave due to gravity. Relative to a GRAVITATIONAL REFERENCE point or equipotential surface of gravity within a given gravity well $g_{y}$, PHOTON A decreases in energy $\varepsilon_{\lambda y}$ and frequency $f_{\lambda y}$ as it propagates through decreasing gravity $g_{-\Delta y}$. Likewise, PHOTON B increases in energy $\mathcal{E}_{\lambda y}$ and frequency $f_{\lambda y}$ as it propagates through increasing gravity $g_{+\Delta y}$. This effect was demonstrated in the Pound, Rebka and Snider experiment, which used Mossbauer spectroscopy to measure the electromagnetic gravitational RED SHIFT (or BLUE SHIFT) of 14.4 keV gamma rays emitted from $\mathrm{Fe}^{57}$ through a vertical distance of 22.6 m . Using the difference form of fluctuating energy $\Delta \varepsilon_{\lambda}$ of an electromagnetic wave propagating through gravity well $g_{y}$ and the gamma rays emitted upward, the RED SHIFT was within one percent (1\%) of this result,

$$
\begin{gather*}
\text { SHIFT }=\frac{\Delta \varepsilon_{\lambda}}{\varepsilon_{\lambda}}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}  \tag{211}\\
\text { SHIFT }=\frac{\left(9.806581 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(6.3781226 \times 10^{6} \mathrm{~m}\right)-\left(9.806650 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(6.3781000 \times 10^{6} \mathrm{~m}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}} \tag{212}
\end{gather*}
$$

$$
\text { SHIFT }=-2.465961 \times 10^{-15}
$$

And with the gamma rays emitted downward, the BLUE SHIFT was,

$$
\begin{gather*}
\text { SHIFT }=\frac{\left(9.806650 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(6.3781000 \times 10^{6} \mathrm{~m}\right)-\left(9.806581 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(6.3781226 \times 10^{6} \mathrm{~m}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{214}\\
\text { SHIFT }=2.465961 \times 10^{-15} \tag{215}
\end{gather*}
$$

The difference form and exponential form of the natural relativistic electromagnetic energy $\varepsilon_{\lambda y}$ at position $\pm \Delta y$ is,

$$
\begin{equation*}
\varepsilon_{\lambda y_{1}}=\varepsilon_{\lambda y_{0}} \pm \Delta \varepsilon_{\lambda y_{0}}=\varepsilon_{\lambda y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=\varepsilon_{\lambda y_{0}} e^{\left(\frac{g_{y_{1} y_{1}-g_{y_{0}} y_{0}}}{c^{2}}\right)} \tag{216}
\end{equation*}
$$

Since the energy $\varepsilon_{\lambda}$ of a single photon is,

$$
\begin{equation*}
\varepsilon_{\lambda}=h f_{\lambda} \tag{217}
\end{equation*}
$$

Then, the Planck's constant $h$ is,

$$
\begin{equation*}
h=\frac{\varepsilon_{\lambda}}{f_{\lambda}} \tag{218}
\end{equation*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of frequency $\Delta f_{\lambda} / f_{\lambda}$ of an electromagnetic wave propagating $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta f_{\lambda}}{f_{\lambda}}=\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{219}
\end{equation*}
$$

The difference form and exponential form of the natural relativistic electromagnetic frequency $f_{\lambda y}$ at position $\pm \Delta y$ is,

$$
\begin{equation*}
f_{\lambda y_{1}}=f_{\lambda y_{0}} \pm \Delta f_{\lambda y_{0}}=f_{\lambda y_{0}}\left(1 \pm \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=f_{\lambda y_{0}} e^{\left(\frac{g_{y_{1} y_{1}-g_{y_{0}} y_{0}}^{c^{2}}}{c^{2}}\right)} \tag{220}
\end{equation*}
$$

So, the natural relativistic Planck's constant $h_{y}$ ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
h_{y_{1}}=\frac{\varepsilon_{\lambda y_{1}}}{f_{\lambda y_{1}}}=\frac{\varepsilon_{\lambda y_{0}} e^{\left(\frac{g_{y_{1} y_{1}-g_{y_{0}} y_{0}}^{c^{2}}}{c^{2}}\right.}}{f_{\lambda y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}}=\frac{\boldsymbol{\varepsilon}_{\lambda}}{f_{\lambda}}=h \tag{221}
\end{equation*}
$$

Therefore, the natural relativistic Planck's constant $h_{y}$ is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
h_{y}=6.6260755 \times 10^{-34} \text { Joule } \cdot \text { sec } \tag{222}
\end{equation*}
$$

The speed of light $c$ is,

$$
\begin{equation*}
c=\lambda f_{\lambda} \tag{223}
\end{equation*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of the wavelength $\Delta \lambda / \lambda$ of an electromagnetic wave displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta \lambda}{\lambda}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{224}
\end{equation*}
$$

The natural relativistic wavelength $\lambda_{y}$ at position $\pm \Delta y$ is,

$$
\begin{equation*}
\lambda_{y_{1}}=\lambda_{y_{0}} \mp \Delta \lambda_{y_{0}}=\lambda_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=\lambda_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{225}
\end{equation*}
$$

So, natural relativistic the speed of light $c_{y}$ ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
c_{y_{1}}=\lambda_{y_{1}} f_{\lambda y_{1}}=\lambda_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} f_{\lambda y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}=\lambda f_{\lambda}=c \tag{226}
\end{equation*}
$$

Therefore, the natural relativistic speed of light $c_{y}$ is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
c_{y}=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec} \tag{227}
\end{equation*}
$$

## HOW GRAVITY AFFECTS THE PERMEABLILITY OF SPACE-TIME MEDIA

The permeability $\mu_{0}$ of space-time media is given as,

$$
\begin{equation*}
\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m} \tag{228}
\end{equation*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of inductance $\Delta L / L$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta L}{L}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{229}
\end{equation*}
$$

The inductance $L_{y}$ of space-time media at position $\pm \Delta y$ is,

$$
\begin{equation*}
L_{y_{1}}=L_{y_{0}} \mp \Delta L_{y_{0}}=L_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=L_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{230}
\end{equation*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of length $\Delta \downarrow / \downarrow$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta \swarrow}{\swarrow}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{231}
\end{equation*}
$$

The length $\mathcal{L}_{y}$ of space-time media at position $\pm \Delta y$ is,

$$
\begin{equation*}
\mathscr{L}_{y_{1}}=\mathscr{L}_{y_{0}} \mp \Delta \mathscr{L}_{y_{0}}=\mathscr{L}_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=\mathscr{L}_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{232}
\end{equation*}
$$

So, the natural relativistic permeability $\mu_{y}$ of space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
\mu_{y_{1}}=\frac{L_{y_{1}}}{\swarrow_{y_{1}}}=\frac{L_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}}{\left.L_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right.}\right)}=\frac{L}{\swarrow}=\mu_{0} \tag{233}
\end{equation*}
$$

Therefore, the natural relativistic permeability $\mu_{y}$ of space-time media is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
\mu_{y}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m} \tag{234}
\end{equation*}
$$

## HOW GRAVITY AFFECTS THE PERMITTIVITY OF SPACE-TIME MEDIA

The permittivity $\varepsilon_{0}$ of space-time media is given as,

$$
\begin{equation*}
\varepsilon_{0}=8.85419 \times 10^{-12} \mathrm{~F} / \mathrm{m} \tag{235}
\end{equation*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of capacitance $\Delta C / C$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta C}{C}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{236}
\end{equation*}
$$

The capacitance $C_{y}$ of space-time media at position $\pm \Delta y$ is,

$$
\begin{equation*}
\left.C_{y_{1}}=C_{y_{0}} \mp \Delta C_{y_{0}}=C_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=C_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right.}\right) \tag{237}
\end{equation*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of length $\Delta \downarrow / \downarrow$ displaced $\Delta y=y_{0}-y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{equation*}
\frac{\Delta \swarrow}{\swarrow}=-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}} \tag{238}
\end{equation*}
$$

The length $\mathcal{L}_{y}$ of space-time media at position $\pm \Delta y$ is,

$$
\begin{equation*}
\mathscr{L}_{y_{1}}=\mathscr{L}_{y_{0}} \mp \Delta \mathscr{L}_{y_{0}}=\mathscr{L}_{y_{0}}\left(1 \mp \frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)=\mathscr{L}_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{239}
\end{equation*}
$$

So, the natural relativistic permittivity $\varepsilon_{y}$ of space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
\varepsilon_{y_{1}}=\frac{C_{y_{1}}}{\mathcal{L}_{y_{1}}}=\frac{C_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}}{\mathcal{L}_{y_{0}} e^{\left(-\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}}=\frac{C}{\swarrow}=\varepsilon_{0} \tag{240}
\end{equation*}
$$

Therefore, the natural relativistic permittivity $\varepsilon_{y}$ of space-time media is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
\varepsilon_{y}=8.85419 \times 10^{-12} \mathrm{~F} / \mathrm{m} \tag{241}
\end{equation*}
$$

## HOW GRAVITY AFFECTS THE VIRTUAL RESISTANCE OF SPACE-TIME MEDIA

The space-time media has virtual resistance or impedance $Z_{0}$, and therefore, isn't capable of absorbing or dissipating electromagnetic energy. Its REAL resistance is infinite or, $R_{0}=\infty$. This media serves to impede the propagation of light and the motion of matter and is calculated as,

$$
\begin{equation*}
Z_{0}=\sqrt{\frac{\mu_{0}}{\varepsilon_{0}}} \tag{242}
\end{equation*}
$$

Since it has been shown the permeability $\mu_{y}$ and permittivity $\varepsilon_{y}$ of space-time media are invariant between equipotential surfaces of gravity, it follows the natural relativistic impedance $Z_{y}$ of space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
Z_{y_{1}}=\sqrt{\frac{\mu_{y_{1}}}{\varepsilon_{y_{1}}}}=Z_{0} \tag{243}
\end{equation*}
$$

Therefore, the natural relativistic impedance $Z_{y}$ of space-time media is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
Z_{y}=376.730 \Omega \tag{244}
\end{equation*}
$$

## HOW GRAVITY AFFECTS THE SPEED OF LIGHT

The speed of light $c$ between space-time media is,

$$
\begin{equation*}
c=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}} \tag{245}
\end{equation*}
$$

Since it has been shown the permeability $\mu_{y}$ and permittivity $\varepsilon_{y}$ of space-time media are invariant between equipotential surfaces of gravity, it follows the natural relativistic speed of light $c_{y}$ through space-time media ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
c_{y_{1}}=\frac{1}{\sqrt{\mu_{y_{1}} \varepsilon_{y_{1}}}}=c \tag{246}
\end{equation*}
$$

Therefore, the natural relativistic speed of light $c_{y}$ through space-time media is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
c_{y}=2.997924 \times 10^{8} \mathrm{~m} / \mathrm{sec} \tag{247}
\end{equation*}
$$

## HOW GRAVITY AFFECTS BOLTZMANN'S CONSTANT

The Boltzmann's Constant $k$ is given as,

$$
\begin{equation*}
k=\frac{R}{N_{0}} \tag{248}
\end{equation*}
$$

Since the Ideal Gas Constant $R_{y}$ and Avogadro's Number $N_{y}$ are invariant between equipotential surfaces of gravity, it follows the natural relativistic Boltzmann's Constant $k_{y}$ ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
k_{y_{1}}=\frac{R_{y_{1}}}{N_{y_{1}}}=k \tag{249}
\end{equation*}
$$

Therefore, the natural relativistic Boltzmann's Constant $k_{y}$ is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
k_{y}=1.380658 \times 10^{-23} \text { Joules } /{ }^{\circ} \mathrm{K} \tag{250}
\end{equation*}
$$

## HOW GRAVITY AFFECTS AN ELECTRIC CHARGE

A fundamental electric charge $q$ is given as,

$$
\begin{equation*}
q=\frac{f}{\varepsilon} \tag{251}
\end{equation*}
$$

The electric force $f_{y}$ increases with the square of a decreasing distance, and the electric field $\varepsilon_{y}$ also increases with the square of a decreasing distance at position $+d y$. Likewise, the electric force $f_{y}$ decreases with the square of a
increasing distance, and the electric field $\mathcal{\delta}_{y}$ also decreases with the square of a increasing distance at position $-d y$, it follows the natural relativistic electric charge $q_{y}$ ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
q_{y_{1}}=\frac{f_{y_{1}}}{\varepsilon_{y_{1}}}=q \tag{252}
\end{equation*}
$$

Therefore, the natural relativistic electric charge $q_{y}$ is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
q_{y}=1.60217733 \times 10^{-19} \text { Coul } \tag{253}
\end{equation*}
$$

## HOW GRAVITY AFFECTS THE FINE STRUCTURE CONSTANT

The Fine Structure Constant $\alpha$ is,

$$
\begin{equation*}
\alpha=\frac{q^{2}}{2 \varepsilon_{0} h c} \tag{254}
\end{equation*}
$$

Since an electric charge $q_{y}$, the speed of light $c_{y}$, the permittivity $\varepsilon_{y}$, and Planck's constant $h_{y}$ are invariant between equipotential surfaces of gravity, it follows the natural relativistic Fine Structure Constant $\alpha_{y}$ ranging from position $-\Delta y$ to $+\Delta y$ evaluates to unity gain or,

$$
\begin{equation*}
\alpha_{y_{1}}=\frac{q_{y_{1}}{ }^{2}}{2 \varepsilon_{y_{1}} h_{y_{1}} c_{y_{1}}}=\alpha \tag{255}
\end{equation*}
$$

Therefore, the natural relativistic Fine Structure Constant $\alpha_{y}$ is invariant between equipotential surfaces of gravity or,

$$
\begin{equation*}
\alpha_{y}=7.29738 \times 10^{-3}=\frac{1}{137.0356} \tag{256}
\end{equation*}
$$

## A TYPICAL ELECTROMAGNETIC WAVE



FIGURE 17. Propagation of electromagnetic wave in flat space-time.



FIGURE 18. Typical $\mathcal{B}$ and $\mathcal{E}$ Fields.

## GRAVITATIONAL BLUE SHIFTING OF AN ELECTROMAGNETIC WAVE



FIGURE 19. Propagation of electromagnetic wave in rarefied space-time.



FIGURE 20. Increasing magnitude and frequency of $\mathcal{B}$ and $\mathcal{E}$ Fields by gravitational function.

## GRAVITATIONAL RED SHIFTING OF AN ELECTROMAGNETIC WAVE



FIGURE 21. Propagation of electromagnetic wave in compressing space-time.


FIGURE 22. Decreasing magnitude and frequency of $\mathcal{B}$ and $\mathcal{E}$ Fields by gravitational function.


FIGURE 23. A Global Positioning Satellite.
Example 5. A Global Positioning Satellite (GPS) transmits an electromagnetic signal at a frequency $f_{\lambda S A T}$ of $\sim 10.23 \mathrm{MHz}$ down to the Earth from an altitude of $20,186.8 \mathrm{~km}$, and has an orbital velocity of $3.874 \mathrm{~km} / \mathrm{sec}$. The natural relativistic BLUE SHIFT due to gravity and the special relativistic RED SHIFT due to velocity changes the frequency of this transmitted signal. So, given the corrected transmitted frequency $f_{\lambda S A T}$, compute the BLUE SHIFT and RED SHIFT such that a ground-based receiver will read a signal $f_{\lambda_{R X}}$ that is precisely 10230000.000000 Hz . The signal frequency of the satellite is adjustable down to $1 \mu \mathrm{~Hz}$.

So, given,
Altitude of satellite $\Delta y=20.1868 \times 10^{6} \mathrm{~m}$
Orbital velocity $v_{x}=3.874 \times 10^{3} \mathrm{~m} / \mathrm{sec}$
Corrected frequency of satellite $f_{\lambda S A T}=10229999.995444 \mathrm{~Hz}$
Receiver located on surface of Earth $y_{1}=6.3781 \times 10^{6} \mathrm{~m}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The initial radius $y_{0}$ of the satellite above the Earth is,

$$
\begin{equation*}
y_{0}=y_{1}+\Delta y=\left(6.3781 \times 10^{6} \mathrm{~m}\right)+\left(20.1868 \times 10^{6} \mathrm{~m}\right)=26.5649 \times 10^{6} \mathrm{~m} \tag{257}
\end{equation*}
$$

The acceleration due to gravity at altitude $y_{0}=26.5649 \times 10^{6} \mathrm{~m}$ above the Earth is,

$$
\begin{equation*}
g_{y_{0}}=\frac{G M_{E}}{y_{0}{ }^{2}}=\frac{\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(26.5649 \times 10^{6} \mathrm{~m}\right)^{2}}=0.5653 \mathrm{~m} / \mathrm{sec}^{2} \tag{258}
\end{equation*}
$$

The acceleration due to gravity at altitude $y_{1}=7.529 \times 10^{6} \mathrm{~m}$ above the Earth is,

$$
\begin{equation*}
g_{y_{1}}=\frac{G M_{E}}{y_{1}{ }^{2}}=\frac{\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)^{2}}=9.80665 \mathrm{~m} / \mathrm{sec}^{2} \tag{259}
\end{equation*}
$$

Given the exponential solution of the natural relativistic frequency model, the gravitational BLUE SHIFTED frequency is,

$$
\begin{gather*}
f_{\lambda y_{1}}=f_{\lambda y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}=(10229999.995444 \mathrm{~Hz}) e^{\left(\frac{\left(9.80665 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(6.3781 \times 10^{6} \mathrm{~m}\right)-\left(0.5653 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(26.5649 \times 10^{6} \mathrm{~m}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}\right)}  \tag{260}\\
f_{\lambda y_{1}}=10230000.000854 \mathrm{~Hz}=f_{\lambda} \tag{261}
\end{gather*}
$$

Given the exponential solution of the special relativistic frequency model, the RED SHIFTED frequency of the BLUE SHIFTED frequency computed above is,

$$
\begin{align*}
& f_{\lambda v}=f_{\lambda} e^{\left(-\frac{v_{x}^{2}}{2 c^{2}}\right)}=(10230000.000854 \mathrm{~Hz}) e^{\left(-\frac{\left(3.874 \times 10^{3} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99993 \times 10^{8} \mathrm{~m} / \mathrm{sc}\right)^{2}}\right)}  \tag{262}\\
& f_{\lambda v}=10230000.000000 \mathrm{~Hz}=f_{\lambda R X} \tag{263}
\end{align*}
$$

So, a ground-based receiver will read a signal that is precisely 10230000.000000 Hz with a satellite frequency $f_{\lambda S A T}$ given above.


FIGURE 24. A constellation of 24 Global Positioning Satellites (GPS) orbiting the Earth.

## GRAVITOMAGNETIC THEORY



FIGURE 25. The magnetic induction produced by a positive current element.
A constant positive electric current $I$ must create a stable magnetic field $B$ around a wire. This stable field is due to the flow of electric current shown above. The change of magnetic induction $d B$ at a fixed point $P$ produced by a current element $d x$ is calculated using the Biot-Savart's Law,

$$
\begin{equation*}
d B=\frac{\mu_{0} I}{4 \pi} \frac{d x \times r}{r^{3}} \tag{264}
\end{equation*}
$$

Or,

$$
\begin{equation*}
d B=\frac{\mu_{0} I}{4 \pi} \frac{\sin (\theta) d x}{r^{2}} \tag{265}
\end{equation*}
$$

Since charge $q$ is quantized in a single electron $e^{-}$then, electric current $I$ is defined as quantity $N$ of charges $e^{-}$ passing a fixed point per change of time $d t$ or,

$$
\begin{equation*}
I=\dot{q}=\frac{d q}{d t}=\frac{d\left(N e^{-}\right)}{d t} \tag{266}
\end{equation*}
$$

And velocity $v_{x}$ of an electron passing a fixed point is defined as change of distance $d x$ per change of time $d t$ or,

$$
\begin{equation*}
v_{x}=\dot{x}=\frac{d x}{d t} \tag{267}
\end{equation*}
$$

Then, the electric current $I$ is redefined as,

$$
\begin{equation*}
I=v_{x} \frac{d\left(N e^{-}\right)}{d x} \tag{268}
\end{equation*}
$$

So, the change of magnetic induction $d B$ at a fixed point $P$ produced by quantity $N$ of charges $e^{-}$moving at velocity $v_{x}$ is,

$$
\begin{equation*}
d B=\frac{\mu_{0} v_{x}}{4 \pi} \frac{\sin (\theta) d\left(N e^{-}\right)}{r^{2}} \tag{269}
\end{equation*}
$$

To find the magnetic induction $B$ produced by a single electron at point $P$ when $\theta=90^{\circ}$ and $N=1$, then integrate,

$$
\begin{equation*}
B=\int d B=\frac{\mu_{0} e^{-} v_{x}}{4 \pi r^{2}} \tag{270}
\end{equation*}
$$

The total energy density $u_{B}$ of magnetic field $B$ contained within volume $v$ is,

$$
\begin{equation*}
u_{B}=\frac{U_{B}}{v}=\frac{B^{2}}{2 \mu_{0}} \tag{271}
\end{equation*}
$$

Therefore, the total field energy $U_{B}$ of magnetic field $B$ contained within volume $V$ is,

$$
\begin{equation*}
U_{B}=\frac{B^{2}}{2 \mu_{0}} \vartheta=\frac{\mu_{0}\left(e^{-}\right)^{2} v_{x}^{2}}{32 \pi^{2} r^{4}} \vartheta \tag{272}
\end{equation*}
$$

The change of magnetic field energy $d U_{B}$ contained within a change of volume $d v$ is,

$$
\begin{equation*}
d U_{B}=\frac{B^{2}}{2 \mu_{0}} d v=\frac{\mu_{0}\left(e^{-}\right)^{2} v_{x}^{2}}{32 \pi^{2} r^{4}} d v \tag{273}
\end{equation*}
$$

The total energy $E_{M}$ contained within matter is,

$$
\begin{equation*}
E_{M}=M c^{2} \tag{274}
\end{equation*}
$$

Equate total magnetic field energy $U_{B}$ to the total energy $E_{M}$ contained within matter,

$$
\begin{equation*}
U_{B}=E_{M} \tag{275}
\end{equation*}
$$

So, the change of magnetic field energy $d U_{B}$ is,

$$
\begin{equation*}
d U_{B}=d M_{B} c^{2} \tag{276}
\end{equation*}
$$

Therefore, the fluctuating magnetic mass $d M_{B}$ contained within a change of volume $d v$ is,

$$
\begin{equation*}
d M_{B}=\frac{d U_{B}}{c^{2}}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{32 \pi^{2} r^{4}} \frac{v_{x}^{2}}{c^{2}} d v \tag{277}
\end{equation*}
$$



FIGURE 26. The volume $v$ of an electron is modeled as a hollow spheroid.
Since the energy of an electron is finite, no field component can be present at its' center. So, the volume of an electron is modeled as a hollow spheroid,

$$
\begin{equation*}
V=2 \pi \int_{0}^{\pi} \sin (\omega) d \omega \int_{r_{\min }}^{r_{\max }} r^{2} d r=4 \pi \int_{r_{\min }}^{r_{\max }} r^{2} d r \tag{278}
\end{equation*}
$$

The fluctuating magnetic mass $d M_{B}$ contained within a change of volume $d \boldsymbol{v}$ of a hollow spheroid is,

$$
\begin{equation*}
d M_{B}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{32 \pi^{2} r^{4}} \frac{v_{x}^{2}}{c^{2}} d\left(4 \pi \int_{r_{\min }}^{r_{\max }} r^{2} d r\right)=\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi} \frac{v_{x}^{2}}{c^{2}} d\left(\int_{r_{\min }}^{r_{\max }} \frac{1}{r^{2}} d r\right) \tag{279}
\end{equation*}
$$

Given the radius of a fluctuating magnetic mass $d M_{e}$ ranging from a classic electron radius $r_{e}$ to infinity, or $r_{e} \leq r \leq \infty$, the derivative form of a moving electron is,

$$
\begin{equation*}
d M_{B}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi} \frac{v_{x}^{2}}{c^{2}} \int_{r_{e}}^{\infty} \frac{1}{r^{2}} d r \tag{280}
\end{equation*}
$$

The fluctuating magnetic mass $d M_{B}$ is,

$$
\begin{equation*}
d M_{B}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi r_{e}} \frac{v_{x}^{2}}{c^{2}} \tag{281}
\end{equation*}
$$

So, given the rest mass of an electron $M_{e}$, the difference form of the special relativistic mass $M_{e v}$ of an electron moving at velocity $v_{x}$ is

$$
\begin{equation*}
M_{e v}=\gamma_{S R} M_{e}=M_{e} \pm d M_{e}=M_{e}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right) \tag{282}
\end{equation*}
$$

By equating the fluctuating magnetic mass $d M_{B}$ to the special relativistic mass $d M_{e}$,

$$
\begin{equation*}
\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi r_{e}} \frac{v_{x}^{2}}{c^{2}} \Leftrightarrow \frac{M_{e}}{2} \frac{v_{x}^{2}}{c^{2}} \tag{283}
\end{equation*}
$$

The equation reduces to,

$$
\begin{equation*}
\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi r_{e}} \Leftrightarrow \frac{M_{e}}{2} \tag{284}
\end{equation*}
$$

So, given,
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Classic electron radius $r_{e}=2.817941 \times 10^{-15} \mathrm{~m}$
Rest mass of an electron $M_{e}=9.109390 \times 10^{-31} \mathrm{~kg}$

$$
\begin{equation*}
\frac{\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right)\left(1.602177 \times 10^{-19} \mathrm{C}\right)^{2}}{8 \pi\left(2.817941 \times 10^{-15} \mathrm{~m}\right)} \Leftrightarrow \frac{9.109390 \times 10^{-31} \mathrm{~kg}}{2} \tag{285}
\end{equation*}
$$

This shows the fluctuating magnetic mass of a moving electron is identical to the special relativistic mass at any velocity $v_{x}$,

$$
\begin{equation*}
4.554693 \Leftrightarrow 4.554695 \tag{286}
\end{equation*}
$$

Therefore, the fluctuating magnetic mass $d M_{B}$ is the fluctuating mass $d M_{e}$ of an electron,

$$
\begin{equation*}
d M_{B}=d M_{e} \tag{287}
\end{equation*}
$$

So, the magnetic mass $M_{B}$ and the mass $M_{e}$ of the electron is,

$$
\begin{equation*}
M_{B}=M_{e}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{4 \pi r_{e}} \tag{288}
\end{equation*}
$$

And the fluctuating magnetic mass $\Delta M_{B}$ and the fluctuating mass $\Delta M_{e}$ is,

$$
\begin{equation*}
\Delta M_{B}=\Delta M_{e}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi r_{e}} \frac{v_{x}^{2}}{c^{2}}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{4 \pi r_{e}} \frac{v_{x}^{2}}{2 c^{2}}=M_{e} \frac{v_{x}^{2}}{2 c^{2}} \tag{289}
\end{equation*}
$$

So, the velocity of an electron is,

$$
\begin{equation*}
v_{x}=2 c \sqrt{\frac{2 \pi r_{e} \Delta M_{e}}{\mu_{0}\left(e^{-}\right)^{2}}}=c \sqrt{\frac{2 \Delta M_{e}}{M_{e}}} \tag{290}
\end{equation*}
$$

Therefore, if the fluctuating mass is positive, then the velocity of the electron is real. However, if the fluctuating mass is negative, then the velocity is imaginary.


FIGURE 27. The magnetic fluctuating mass.

Now, the difference form of the inertial RED SHIFT (or BLUE SHIFT) of the magnetic mass $\Delta M_{B} / M_{B}$ and the mass $\Delta M_{e} / M_{e}$ of a particle moving at a velocity $v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta M_{B}}{M_{B}}=\frac{\Delta M_{e}}{M_{e}}=\frac{v_{x}^{2}}{2 c^{2}} \tag{291}
\end{equation*}
$$

A particle can move at a real (i.e., time-forward) velocity $v_{x}$, at an imaginary (i.e., time-future) velocity $j v_{x}$, or at a velocity that is a combination of the two. The real and imaginary components are rotated about the temporal axis and therefore, can be described as complex motion. The rotation is given as $0^{\circ} \leq \theta \leq 90^{\circ}$, where the real axis is $\theta=0^{\circ}$ and the imaginary time-future axis is $\theta=90^{\circ}$. The complex number uses the Euler's identity $e^{j \theta}$, which functions as a temporal rotation operator. The complex velocity $v_{x}$ is,

$$
\begin{equation*}
v_{x}=v e^{j \theta}=v \cos \theta+j v \sin \theta \tag{292}
\end{equation*}
$$

Given the rest mass of an electron $M_{e}$ or the classic electron radius $r_{e}$, the difference forms of the special relativistic magnetic mass $M_{e v}$ model of a particle moving at a complex velocity $v_{x}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ are,

$$
\begin{gather*}
M_{e v}=M_{e} \pm \Delta M_{e}=M_{e}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=M_{e} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)}  \tag{293}\\
M_{e v}=M_{e} \pm \Delta M_{e}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{4 \pi r_{e}}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=\frac{\mu_{0}\left(e^{-}\right)^{2}}{4 \pi r_{e}} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{294}
\end{gather*}
$$

Now, apply the new Principle of Equivalence Theorem where the fluctuating magnetic mass of a moving electron is equivalent to natural relativistic mass due to the Earth's gravity well,

$$
\begin{gather*}
\Delta M_{e}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{8 \pi r_{e}} \frac{v_{x}^{2}}{c^{2}}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=M_{e} \frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}=G M_{E} M_{e} \frac{\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}}  \tag{295}\\
v_{x}=\sqrt{2\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}=\sqrt{2 G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{296}
\end{gather*}
$$

The position $y_{1}$ of an electron moving at a velocity $v_{x}$ within Earth's gravity well $g_{Y}$ where $0<y_{1} \leq \infty$ or $-1 \leq \frac{y_{0} v_{x}{ }^{2}}{2 G M_{E}}$ is,

$$
\begin{gather*}
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{v_{x}^{2}}{2}\right)=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{\Delta M_{e} c^{2}}{M_{e}}\right)  \tag{297}\\
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{y_{0}}{1+\frac{y_{0} c^{2} \Delta M_{e}}{G M_{E} M_{e}}} \tag{298}
\end{gather*}
$$

The equivalent maximum complex velocity $v_{x \max }$ at $y_{1}=\infty$ is,

$$
\begin{equation*}
v_{x \max }=\sqrt{-\frac{2 G M_{E}}{y_{0}}} \tag{299}
\end{equation*}
$$

Given the equivalent maximum complex velocity $v_{x \max }$, the minimum gravitational mass $M_{e \min }$ at $y_{1}=\infty$ is,

$$
\begin{equation*}
M_{e \text { min }}=M_{e}\left(1+\frac{v_{x}^{2}}{2 c^{2}}\right)=M_{e}\left(1-\frac{G M_{E}}{y_{0} c^{2}}\right)=M_{e} e^{\left(-\frac{G M_{E}}{y_{0} c^{2}}\right)} \tag{300}
\end{equation*}
$$

The equivalent maximum fluctuating gravitational mass of the electron $\Delta M_{e}$ at $y_{1}=\infty$ is,

$$
\begin{equation*}
\Delta M_{e \max }=M_{e \min }-M_{e}=-\frac{G M_{E} M_{e}}{y_{0} c^{2}} \tag{301}
\end{equation*}
$$

So, the difference form of the gravitational RED SHIFT (or BLUE SHIFT) of the magnetic mass $\Delta M_{B} / M_{B}$ and the mass $\Delta M_{e} / M_{e}$ of a particle displaced a distance $\Delta y$ within a given gravity well $g_{Y}$ is,

$$
\begin{equation*}
\frac{\Delta M_{B}}{M_{B}}=\frac{\Delta M_{e}}{M_{e}}=\frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}} \tag{302}
\end{equation*}
$$

Given the rest mass of an electron $M_{e}$ or the classic electron radius $r_{e}$, the difference forms of the natural relativistic mass $M_{e y_{1}}$ model of a particle displaced a distance $\Delta y$ within a given gravity well $g_{Y}$ are,

$$
\begin{gather*}
M_{e y_{1}}=M_{e} \pm \Delta M_{e}=M_{e}\left(1 \pm \frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}\right)=M_{e} e^{\left(\frac{g_{y_{1} y_{1}}-g_{y_{0}} y_{0}}{c^{2}}\right)}  \tag{303}\\
M_{e y_{1}}=M_{e} \pm \Delta M_{e}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{4 \pi r_{e}}\left(1 \pm \frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}\right)=\frac{\mu_{0}\left(e^{-}\right)^{2}}{4 \pi r_{e}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{304}
\end{gather*}
$$

In summary, Gravitomagnetic Theory shows that a moving electron produces an increase in relativistic mass that extends from its' classic radius $r_{e}$ to infinity, and couples to gravity. This motion can either have a velocity $v_{x}$ or a complex (i.e., time-future) velocity $j v_{x}$. If the velocity is complex, then the electron will exhibit an antigravitational effect, and produce a complex (i.e., time-future) magnetic field $j B$. In addition, the total field energy $U_{B}$ of a complex magnetic field $j B$ contained within a volume $v$ is NEGATIVE.

Example 6. An electron $e^{-}$moving through a wire at a time-forward velocity $v_{x}$ where $\theta=0^{\circ}$ produces a timeforward magnetic induction $B$ at a distance $r$.

Given,
Direction of time is forward $\theta=0^{\circ}$
Velocity of electron $e^{-}$through a wire $v=1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Rest mass of an electron $M_{e}=9.109390 \times 10^{-31} \mathrm{~kg}$
Radius $r=1.0 m$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The time-forward velocity $v_{x}$, where $\theta=0^{\circ}$ is,

$$
\begin{equation*}
v_{x}=v e^{j \theta}=\left(1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right) e^{j 0^{\circ}}=1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec} \tag{305}
\end{equation*}
$$

The time-forward magnetic induction $B$ at distance $r$ is,

$$
\begin{gather*}
B=\frac{\mu_{0} e^{-} v_{x}}{4 \pi r^{2}}=\frac{\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right)\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right)}{4 \pi(1.0 \mathrm{~m})^{2}}  \tag{306}\\
B=1.602177 \times 10^{-28} \mathrm{~T} \tag{307}
\end{gather*}
$$

The POSITIVE fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{308}\\
\Delta M_{e}=5.067782 \times 10^{-52} \mathrm{~kg} \tag{309}
\end{gather*}
$$

## Applying the new Principle of Equivalence Theorem,

$$
\begin{align*}
& y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{310}\\
& y_{1}=6.37809999999490 \times 10^{6} \mathrm{~m} \tag{311}
\end{align*}
$$

So, the equivalent POSITIVE displacement $\Delta y$ is gravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=5.0981 \times 10^{-6} \mathrm{~m} \tag{312}
\end{equation*}
$$

Example 7. An electron $e^{-}$moving through a wire at a time-advanced velocity $v_{x}$ where $0^{\circ}<\theta<90^{\circ}$ produces a time-advanced magnetic induction $B$ at a distance $r$.

Given,
Direction of time is advanced $\theta=45^{\circ}$
Velocity of electron $e^{-}$through a wire $v=1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Rest mass of an electron $M_{e}=9.109390 \times 10^{-31} \mathrm{~kg}$
Radius $r=1.0 \mathrm{~m}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The time-advanced velocity $v_{x}$, where $\theta=45^{\circ}$ is,

$$
\begin{equation*}
v_{x}=v e^{j \theta}=\left(1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right) e^{j 45^{\circ}}=7.071068 \times 10^{-3}+7.071068 j \times 10^{-3} \mathrm{~m} / \mathrm{sec} \tag{313}
\end{equation*}
$$

The time-advanced magnetic induction $B$ at distance $r$ is,

$$
\begin{gather*}
B=\frac{\mu_{0} e^{-} v_{x}}{4 \pi r^{2}}=\frac{\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right)\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(7.071068 \times 10^{-3}+7.071068 \mathrm{j} \times 10^{-3} \mathrm{~m} / \mathrm{sec}\right)}{4 \pi(1.0 \mathrm{~m})^{2}}  \tag{314}\\
B=1.132910 \times 10^{-28}+1.132910 \mathrm{j} \times 10^{-28} \mathrm{~T} \tag{315}
\end{gather*}
$$

The IMAGINARY fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(7.071068 \times 10^{-3}+7.071068 \mathrm{j} \times 10^{-3} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{316}\\
\Delta M_{e}=5.067782 \mathrm{j} \times 10^{-52} \mathrm{~kg} \tag{317}
\end{gather*}
$$

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(7.071068 \times 10^{-3}+7.071068 \mathrm{j} \times 10^{-3} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{318}\\
y_{1}=6.3781 \times 10^{6} \mathrm{~m}-5.0986 \mathrm{j} \times 10^{-6} \mathrm{~m} \tag{319}
\end{gather*}
$$

So, the equivalent IMAGINARY displacement $\Delta y$ is shown to be non-gravitational within the Earth’s gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=5.0986 j \times 10^{-6} \mathrm{~m} \tag{320}
\end{equation*}
$$

The maximum time-future velocity $v_{x \max }$ within the Earth's gravity well is,

$$
\begin{gather*}
v_{x \max }=\sqrt{-\frac{2 G M_{E}}{y_{0}}}=\sqrt{-\frac{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}}  \tag{321}\\
v_{x \max }=1.11846 \mathrm{j} \times 10^{4} \mathrm{~m} / \mathrm{sec} \tag{322}
\end{gather*}
$$

Example 8. An electron $e^{-}$moving through a wire at a time-future velocity $v_{x}$ where $\theta=90^{\circ}$ produces a timefuture magnetic induction $B$ at a distance $r$.

Given,
Direction of time is future $\theta=90^{\circ}$
Velocity of electron $e^{-}$through a wire $v=1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Rest mass of an electron $M_{e}=9.109390 \times 10^{-31} \mathrm{~kg}$
Radius $r=1.0 \mathrm{~m}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The time-future velocity $v_{x}$, where $\theta=90^{\circ}$ is,

$$
\begin{equation*}
v_{x}=v e^{j \theta}=\left(1.0 \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right) e^{j 90^{\circ}}=1.0 j \times 10^{-2} \mathrm{~m} / \mathrm{sec} \tag{323}
\end{equation*}
$$

The time-future magnetic induction $B$ at distance $r$ is,

$$
\begin{gather*}
B=\frac{\mu_{0} e^{-} v_{x}}{4 \pi r^{2}}=\frac{\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right)\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(1.0 \mathrm{j} \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right)}{4 \pi(1.0 \mathrm{~m})^{2}}  \tag{324}\\
B=1.602177 \mathrm{j} \times 10^{-28} \mathrm{~T} \tag{325}
\end{gather*}
$$

The NEGATIVE fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(1.0 j \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{326}\\
\Delta M_{e}=-5.067782 \times 10^{-52} \mathrm{~kg} \tag{327}
\end{gather*}
$$

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}=} \frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(1.0 j \times 10^{-2} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{328}\\
y_{1}=6.37800000000510 \times 10^{6} \mathrm{~m} \tag{329}
\end{gather*}
$$

So, the equivalent NEGATIVE displacement $\Delta y$ is antigravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=-5.0981 \times 10^{-6} \mathrm{~m} \tag{330}
\end{equation*}
$$

The maximum time-future velocity $v_{x \max }$ within the Earth's gravity well is,

$$
\begin{gather*}
v_{x \max }=\sqrt{-\frac{2 G M_{E}}{y_{0}}}=\sqrt{-\frac{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}}  \tag{331}\\
v_{x \max }=1.11846 \mathrm{j} \times 10^{4} \mathrm{~m} / \mathrm{sec} \gg v_{x} \tag{332}
\end{gather*}
$$

## THE RELATIVITY OF ORBITAL SPIN



FIGURE 28. The definition of a rotating space-time interval.
A linear space-time interval is defined as,

$$
\begin{equation*}
d s_{0}^{2}+d r^{2}=c^{2} d t^{2} \tag{333}
\end{equation*}
$$

Since $z_{0}=0$,

$$
\begin{gather*}
r_{0}^{2}=x_{0}^{2}+y_{0}^{2}+z_{0}^{2}=x_{0}^{2}+y_{0}^{2}  \tag{334}\\
d r^{2}=d x^{2}+d y^{2}+d z^{2}=d x^{2}+d y^{2} \tag{335}
\end{gather*}
$$

According to Fock (1964), a rotating space-time interval is defined as,

$$
\begin{gather*}
x=x_{0} \cos \omega t+y_{0} \sin \omega t  \tag{336}\\
y=-x_{0} \sin \omega t+y_{0} \cos \omega t  \tag{337}\\
d s_{0}^{2}=\left(c^{2}-\omega_{0}^{2}\left(x_{0}^{2}+y_{0}^{2}\right)\right) d t^{2}-2 \omega\left(y_{0} d x-x_{0} d y\right) d t-\left(d x^{2}+d y^{2}+d z^{2}\right) \tag{338}
\end{gather*}
$$

This reduces to,

$$
\begin{equation*}
d s_{0}^{2}=\left(c^{2}-r_{0}^{2} \omega_{0}^{2}\right) d t^{2}-2 \omega_{0}\left(y_{0} d x-x_{0} d y\right) d t-d r^{2} \tag{339}
\end{equation*}
$$

## COMPLEX AMPERIAN CURRENTS



FIGURE 29. The complete complex Bohr model of the Hydrogen atom.
In the Bohr model of the Hydrogen atom, electrons move at relativistic speeds in discrete circular orbits around a nucleus. It has been determined the increase in relativistic mass of the electron is in the form of total magnetic field energy produced by the circulating electron. So, as a consequence of this motion, a magnetic induction $B$ is produced at the center of the orbit. If an external magnetic field is applied to this induction, the velocity of the electron becomes complex by partially rotating into the imaginary axis. The velocity of the electron may increase or decrease as a function of the applied external field. If the field opposes the induction $B$, the real velocity will appear to decrease as it rotates into the imaginary axis. If the electron's velocity is fully rotated into the imaginary axis and therefore moving at a time-future velocity $j v_{x}$, a time-future magnetic field $j B$ will emerge from the center. The complete complex Bohr model includes the following characteristic equations shown below. These equations contain the real and imaginary components of a moving electron that is rotated about the temporal axis as complex motion. The rotation is given as $0^{\circ} \leq \theta \leq 90^{\circ}$, where the real axis is $\theta=0^{\circ}$ and the imaginary time-future axis is $\theta=90^{\circ}$. The complex number uses the Euler's identity $e^{j \theta}$, which functions as a temporal rotation operator.

So, given,
Direction of time $\theta$
Frequency of orbit $f$
Permeability of free space $\mu_{0}$
Fundamental charge of an electron $e^{-}$
Classic $1^{\text {st }}$ Bohr orbital radius $r$
Rest mass of an electron $M_{e}$
Speed of light $c$
Gravitational constant $G$
Mean radius of surface of Earth $y_{0}$
Mass of the Earth $M_{E}$
Mean radius of surface of Sun $y_{0}$
Mass of the Sun $M_{\text {SUN }}$

The complex frequency of orbit $f_{0}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is,

$$
\begin{equation*}
f_{0}=f e^{j \theta}=f \cos \theta+j f \sin \theta \tag{340}
\end{equation*}
$$

The complex Amperian Current $i$ of the electron is,

$$
\begin{equation*}
i=e^{-} f_{0} \tag{341}
\end{equation*}
$$

The complex Magnetic Dipole Moment $\mu$ of the electron is,

$$
\begin{equation*}
\mu=\pi r^{2} i=\pi r^{2} e^{-} f_{0} \tag{342}
\end{equation*}
$$

The complex angular velocity $\omega_{0}$ of the electron is,

$$
\begin{equation*}
\omega_{0}=2 \pi f_{0} \tag{343}
\end{equation*}
$$

The complex velocity $v_{x}$ of the electron is,

$$
\begin{equation*}
v_{x}=r \omega_{0}=2 \pi r f_{0} \tag{344}
\end{equation*}
$$

The complex magnetic field $B$ at the center axis of the orbit $z=0$ is,

$$
\begin{gather*}
B=\frac{\mu_{0} r^{2} i}{2\left(r^{2}+z^{2}\right)^{3 / 2}}=\frac{\mu_{0} \mu}{2 \pi\left(r^{2}+z^{2}\right)^{3 / 2}}=\frac{\mu_{0} r^{2} e^{-} f_{0}}{2\left(r^{2}+z^{2}\right)^{3 / 2}}=\frac{\mu_{0} r^{2} e^{-} \omega_{0}}{4 \pi\left(r^{2}+z^{2}\right)^{3 / 2}}=\frac{\mu_{0} r e^{-} v_{x}}{4 \pi\left(r^{2}+z^{2}\right)^{3 / 2}}  \tag{345}\\
B=\frac{\mu_{0} i}{2 r}=\frac{\mu_{0} \mu}{2 \pi r^{3}}=\frac{\mu_{0} e^{-} f_{0}}{2 r}=\frac{\mu_{0} e^{-} \omega_{0}}{4 \pi r}=\frac{\mu_{0} e^{-} v_{x}}{4 \pi r^{2}} \tag{346}
\end{gather*}
$$

The magnetic force $F_{v}$ of the electron $e^{-}$directed upon the nucleus is,

$$
\begin{gather*}
F_{v}=e^{-} v_{x} \times B  \tag{347}\\
F_{v}=\mu_{0} \pi i^{2}=\frac{\mu_{0} \mu^{2}}{\pi r^{4}}=\mu_{0} \pi\left(e^{-}\right)^{2} f_{0}^{2}=\frac{\mu_{0}\left(e^{-}\right)^{2} \omega_{0}^{2}}{4 \pi}=\frac{\mu_{0}\left(e^{-}\right)^{2} v_{x}^{2}}{4 \pi r^{2}} \tag{348}
\end{gather*}
$$

The direction of electron motion $v_{x}$ is such that the magnetic force $F_{v}$ is an attractive force between the electron and the nucleus.

The special relativistic mass $M_{e v}$ of the circulating electron is,

$$
\begin{equation*}
M_{e v}=M_{e} \pm \Delta M_{e}=M_{e}\left(1 \pm \frac{v_{x}^{2}}{2 c^{2}}\right)=M_{e} e^{\left(\frac{v_{x}^{2}}{2 c^{2}}\right)} \tag{349}
\end{equation*}
$$

The change in relativistic mass of the electron $\Delta M_{e}$ is in the form of the change in magnetic field energy $\Delta U_{B}$ produced by the circulating electron.

$$
\begin{equation*}
\Delta U_{B}=\Delta E_{e}=\Delta M_{e} c^{2} \tag{350}
\end{equation*}
$$

The difference form of the inertial RED SHIFT (or BLUE SHIFT) of the mass $\Delta M_{e} / M_{e}$ of an electron moving at a velocity $v_{x}$ is,

$$
\begin{equation*}
\frac{\Delta M_{e}}{M_{e}}=\frac{v_{x}^{2}}{2 c^{2}}=\frac{2 \pi^{2} r^{2} i^{2}}{\left(e^{-}\right)^{2} c^{2}}=\frac{2 \mu^{2}}{\left(e^{-}\right)^{2} c^{2} r^{2}}=\frac{2 \pi^{2} r^{2} f_{0}{ }^{2}}{c^{2}}=\frac{r^{2} \omega_{0}{ }^{2}}{2 c^{2}} \tag{351}
\end{equation*}
$$

So, the complex velocity $v_{x}$ of the electron is,

$$
\begin{equation*}
v_{x}=\frac{2 r}{e^{-}} \sqrt{\frac{\pi F_{v}}{\mu_{0}}}=c \sqrt{\frac{2 \Delta M_{e}}{M_{e}}} \tag{352}
\end{equation*}
$$

The complex Amperian Current $i$ of the electron is,

$$
\begin{equation*}
i=\sqrt{\frac{F_{v}}{\mu_{0} \pi}}=\frac{e^{-} c}{\pi r} \sqrt{\frac{\Delta M_{e}}{2 M_{e}}} \tag{353}
\end{equation*}
$$

The complex Magnetic Dipole Moment $\mu$ of the electron is,

$$
\begin{equation*}
\mu=r^{2} \sqrt{\frac{\pi F_{v}}{\mu_{0}}}=e^{-} c r \sqrt{\frac{\Delta M_{e}}{2 M_{e}}} \tag{354}
\end{equation*}
$$

The complex frequency of orbit $f_{0}$ of the electron is,

$$
\begin{equation*}
f_{0}=\frac{1}{e^{-}} \sqrt{\frac{F_{v}}{\mu_{0} \pi}}=\frac{c}{\pi r} \sqrt{\frac{\Delta M_{e}}{2 M_{e}}} \tag{355}
\end{equation*}
$$

The complex angular velocity $\omega_{0}$ of the electron is,

$$
\begin{equation*}
\omega_{0}=\frac{2}{e^{-}} \sqrt{\frac{\pi F_{v}}{\mu_{0}}}=\frac{c}{r} \sqrt{\frac{2 \Delta M_{e}}{M_{e}}} \tag{356}
\end{equation*}
$$

So, the complex magnetic force $F_{v}$ of the electron $e^{-}$directed upon the nucleus is,

$$
\begin{equation*}
F_{v}=\frac{\mu_{0}\left(e^{-}\right)^{2} c^{2}}{2 \pi r^{2}} \frac{\Delta M_{e}}{M_{e}}=\frac{\mu_{0}\left(e^{-}\right)^{2}}{2 \pi r^{2}} \frac{\Delta E_{e}}{M_{e}}==\frac{\mu_{0}\left(e^{-}\right)^{2}}{2 \pi r^{2}} \frac{\Delta U_{B}}{M_{e}} \tag{357}
\end{equation*}
$$

And the difference form of the inertial RED SHIFT (or BLUE SHIFT) of the mass $\Delta M_{e} / M_{e}$ of an electron is,

$$
\begin{equation*}
\frac{\Delta M_{e}}{M_{e}}=\frac{2 \pi r^{2} F_{v}}{\mu_{0}\left(e^{-}\right)^{2} c^{2}} \tag{358}
\end{equation*}
$$

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=M_{e} \frac{r^{2} \omega_{0}^{2}}{2 c^{2}}=M_{e} \frac{2 \pi^{2} r^{2} f_{0}^{2}}{c^{2}}=M_{e} \frac{2 \pi^{2} r^{2} i^{2}}{\left(e^{-}\right)^{2} c^{2}}=M_{e} \frac{2 \mu^{2}}{\left(e^{-}\right)^{2} c^{2} r^{2}}=M_{e} \frac{2 \pi r^{2} F_{v}}{\mu_{0}\left(e^{-}\right)^{2} c^{2}}  \tag{359}\\
\Delta M_{e}=M_{e} \frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}=G M_{E} M_{e} \frac{\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}} \tag{360}
\end{gather*}
$$

So, the complex velocity $v_{x}$ of the electron is,

$$
\begin{equation*}
v_{x}=\sqrt{2\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}=\sqrt{2 G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{361}
\end{equation*}
$$

The complex angular velocity $\omega_{0}$ of the electron is,

$$
\begin{equation*}
\omega_{0}=\frac{1}{r} \sqrt{2\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}=\frac{1}{r} \sqrt{2 G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{362}
\end{equation*}
$$

The complex frequency of orbit $f_{0}$ of the electron is,

$$
\begin{equation*}
f_{0}=\frac{1}{\pi r} \sqrt{\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{2}}=\frac{1}{\pi r} \sqrt{\frac{G M_{E}}{2}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{363}
\end{equation*}
$$

The complex Amperian Current $i$ of the electron is,

$$
\begin{equation*}
i=\frac{e^{-}}{\pi r} \sqrt{\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{2}}=\frac{e^{-}}{\pi r} \sqrt{\frac{G M_{E}}{2}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{364}
\end{equation*}
$$

The complex Magnetic Dipole Moment $\mu$ of the electron is,

$$
\begin{equation*}
\mu=e^{-} r \sqrt{\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{2}}=e^{-} r \sqrt{\frac{G M_{E}}{2}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{365}
\end{equation*}
$$

The complex magnetic force $F_{v}$ of the electron $e^{-}$directed upon the nucleus is,

$$
\begin{equation*}
F_{v}=\frac{\mu_{0}\left(e^{-}\right)^{2}\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{2 \pi r^{2}}=\frac{\mu_{0}\left(e^{-}\right)^{2} G M_{E}}{2 \pi r^{2}}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right) \tag{366}
\end{equation*}
$$

The equivalent displacement to position $y_{1}$ of an electron moving at a velocity $v_{x}$ within Earth's gravity well $g_{Y}$ where $0<y_{1} \leq \infty$ or $-1 \leq \frac{y_{0} v_{x}^{2}}{2 G M_{E}}$ is,

$$
\begin{gather*}
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{v_{x}^{2}}{2}\right)=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}  \tag{367}\\
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{r^{2} \omega_{0}^{2}}{2}\right)=\frac{y_{0}}{1+\frac{y_{0} r^{2} \omega_{0}^{2}}{2 G M_{E}}}  \tag{368}\\
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+2 \pi^{2} r^{2} f_{0}^{2}\right)=\frac{y_{0}}{1+\frac{2 \pi^{2} y_{0} r^{2} f_{0}^{2}}{G M_{E}}}  \tag{369}\\
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{2 \pi^{2} r^{2} i^{2}}{\left(e^{-}\right)^{2}}\right)=\frac{y_{0}}{1+\frac{2 \pi^{2} y_{0} r^{2} i^{2}}{\left(e^{-}\right)^{2} G M_{E}}}  \tag{370}\\
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{2 \mu^{2}}{\left(e^{-}\right)^{2} r^{2}}\right)=\frac{y_{0}}{1+\frac{2 y_{0} \mu^{2}}{\left(e^{-}\right)^{2} r^{2} G M_{E}}}  \tag{371}\\
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{2 \pi r^{2} F_{v}}{\mu_{0}\left(e^{-}\right)^{2}}\right)=\frac{y_{0}}{1+\frac{2 \pi r^{2} y_{0} F_{v}}{\mu_{0}\left(e^{-}\right)^{2} G M_{E}}} \tag{372}
\end{gather*}
$$

The difference form of the gravitational RED SHIFT (or BLUE SHIFT) of the mass $\Delta M_{e} / M_{e}$ of a particle displaced a distance $\Delta y$ within Earth's gravity well $g_{Y}$ is,

$$
\begin{equation*}
\frac{\Delta M_{e}}{M_{e}}=\frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}=\frac{G M_{E}}{c^{2}}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right) \tag{373}
\end{equation*}
$$

Given the rest mass of an electron $M_{e}$, the difference forms of the natural relativistic mass $M_{e y_{1}}$ model of a particle displaced a distance $\Delta y$ within Earth's gravity well $g_{Y}$ are,

$$
\begin{equation*}
M_{e y_{1}}=M_{e} \pm \Delta M_{e}=M_{e}\left(1 \pm \frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}\right)=M_{e} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)} \tag{374}
\end{equation*}
$$

$$
\begin{equation*}
M_{e y_{1}}=M_{e} \pm \Delta M_{e}=M_{e}\left(1 \pm \frac{G M_{E}}{c^{2}}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)\right)=M_{e} e^{\left(\frac{G M_{E}}{c^{2}}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)\right)} \tag{375}
\end{equation*}
$$

In summary, if the real component of the magnetic field is cancelled, or $B=0$, due to an externally applied magnetic field $B_{E X T}$, the velocity of the circulating electron is complex, or $j v_{x}$ and a complex magnetic field $j B$ emerges. This complex magnetic field is believed to be present in the Aharonov-Bohm Experiment, which affected the flow of electrons. The complex Amperian Current uses the temporal rotation operator or Euler's identity $e^{j \theta}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$. As the motion of an electron rotates from real to imaginary, or $\theta \rightarrow 90^{\circ}$, the electrons special relativistic mass $\Delta M_{e}$ in the form of magnetic field energy $\Delta U_{B}$ decreases. In addition, the electrons rest mass $M_{e}$ as well as its charge is invariant during acceleration or deceleration.


FIGURE 30. The time-forward Bohr model of the Hydrogen atom.
Example 9. In the time-forward Bohr model of the Hydrogen atom the electron $e^{-}$circulates around the nucleus at a relativistic velocity $v_{x}$ as shown above. This creates a magnetic induction $B$ emerging from the center of the nucleus.

So, given,
Direction of time is forward $\theta=0^{\circ}$
Frequency of orbit $f=6.8 \times 10^{15} \mathrm{~Hz}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Classic $1^{\text {st }}$ Bohr orbital radius $r=5.291773 \times 10^{-11} \mathrm{~m}$
Rest mass of an electron $M_{e}=9.1093897 \times 10^{-31} \mathrm{~kg}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mean radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$

Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The time-forward frequency of orbit $f_{0}$, where $\theta=0^{\circ}$ is,

$$
\begin{equation*}
f_{0}=f e^{j \theta}=\left(6.8 \times 10^{15} \mathrm{~Hz}\right) e^{j 0^{\circ}}=6.8 \times 10^{15} \mathrm{~Hz} \tag{376}
\end{equation*}
$$

The time-forward Amperian Current $i$ is,

$$
\begin{equation*}
i=e^{-} f_{0}=\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(6.8 \times 10^{15} \mathrm{~Hz}\right)=1.089481 \times 10^{-3} \mathrm{Amps} \tag{377}
\end{equation*}
$$

The time-forward magnetic field $B$ at the center axis of the orbit $z=0$ is,

$$
\begin{gather*}
B=\mu_{0} \frac{r^{2} i}{2\left(r^{2}+z^{2}\right)^{3 / 2}}=\mu_{0} \frac{r^{2} e^{-} f_{0}}{2\left(r^{2}+z^{2}\right)^{3 / 2}}  \tag{378}\\
B=\mu_{0} \frac{i}{2 r}=\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right) \frac{\left(1.089481 \times 10^{-3} \mathrm{Amps}\right)}{2\left(5.291773 \times 10^{-11} \mathrm{~m}\right)}=12.935946 \mathrm{~T} \tag{379}
\end{gather*}
$$

The time-forward angular velocity $\omega_{0}$ of the electron $e^{-}$is,

$$
\begin{equation*}
\omega_{0}=2 \pi f_{0}=2 \pi\left(6.8 \times 10^{15} \mathrm{~Hz}\right)=4.272566 \times 10^{16} \mathrm{~Hz} \tag{380}
\end{equation*}
$$

The time-forward velocity $v_{x}$ of the electron $e^{-}$is,

$$
\begin{equation*}
v_{x}=r \omega_{0}=\left(5.291773 \times 10^{-11} \mathrm{~m}\right)\left(4.272566 \times 10^{16} \mathrm{~Hz}\right)=2.260945 \times 10^{6} \mathrm{~m} / \mathrm{sec} \tag{381}
\end{equation*}
$$

The time-forward magnetic force $F_{v}$ of the electron $e^{-}$directed upon the nucleus is,

$$
\begin{gather*}
F_{v}=e^{-} v_{x} B=\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(2.260945 \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)(12.935946 \mathrm{~T})  \tag{382}\\
F_{v}=4.685962 \times 10^{-12} \mathrm{~N} \tag{383}
\end{gather*}
$$

The direction of electron motion $v_{x}$ is such that the magnetic force $F_{v}$ is always an attractive force between the electron and the nucleus.

The POSITIVE fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{equation*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(2.260945 \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}=2.590585 \times 10^{-35} \mathrm{~kg} \tag{384}
\end{equation*}
$$

The increased special relativistic mass $M_{v}$ of the electron $e^{-}$is,

$$
\begin{equation*}
M_{v}=M_{e}+\Delta M_{e}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right)+\left(2.590585 \times 10^{-35} \mathrm{~kg}\right)=9.109649 \times 10^{-31} \mathrm{~kg} \tag{385}
\end{equation*}
$$

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(2.260945 \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{386}\\
y_{1}=156.0784 \mathrm{~m} \tag{387}
\end{gather*}
$$

So, the equivalent POSITIVE displacement $\Delta y$ is gravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=6.377944 \times 10^{6} \mathrm{~m} \tag{388}
\end{equation*}
$$



FIGURE 31. The time-advanced Bohr model of the Hydrogen atom.
Example 10. In the time-advanced Bohr model of the Hydrogen atom, the real magnetic field created by an electron circulating at relativistic speeds is partially cancelled by an externally applied magnetic field $B_{E X T}$. The electron reacts by rotating its velocity into the imaginary axis as shown above. As a consequence of this complex velocity $v e^{j 45^{\circ}}$, a complex magnetic field $B e^{j 45^{\circ}}$ emerges.

So, given,
Direction of time $\theta=45^{\circ}$
Frequency of orbit $f=6.8 \times 10^{15} \mathrm{~Hz}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Classic $1^{\text {st }}$ Bohr orbital radius $r=5.291773 \times 10^{-11} \mathrm{~m}$
Rest mass of an electron $M_{e}=9.1093897 \times 10^{-31} \mathrm{~kg}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}$

Mean radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$
Mean radius of surface of Sun $y_{0}=6.96 \times 10^{8} \mathrm{~m}$
Mass of the Sun $M_{\text {SUN }}=1.98892 \times 10^{30} \mathrm{~kg}$
The time-advanced frequency of orbit $f_{0}$, where $\theta=45^{\circ}$ is,

$$
\begin{equation*}
f_{0}=f e^{j \theta}=\left(6.8 \times 10^{15} \mathrm{~Hz}\right) e^{j 45^{\circ}}=4.808326 \times 10^{15}+4.808326 j \times 10^{15} \mathrm{~Hz} \tag{389}
\end{equation*}
$$

The time-advanced Amperian Current $i$ is,

$$
\begin{gather*}
i=e^{-} f_{0}=\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(4.808326 \times 10^{15}+4.808326 j \times 10^{15} \mathrm{~Hz}\right)  \tag{390}\\
i=7.703791 \times 10^{-4}+7.703791 j \times 10^{-4} \mathrm{Amps} \tag{391}
\end{gather*}
$$

The time-advanced magnetic field $B$ at the center axis of the orbit $z=0 m$ is,

$$
\begin{gather*}
B=\mu_{0} \frac{r^{2} i}{2\left(r^{2}+z^{2}\right)^{3 / 2}}=\mu_{0} \frac{r^{2} e^{-} f_{0}}{2\left(r^{2}+z^{2}\right)^{3 / 2}}  \tag{392}\\
B=\mu_{0} \frac{i}{2 r}=\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right) \frac{\left(7.703791 \times 10^{-4}+7.703791 j \times 10^{-4} \mathrm{Amps}\right)}{2\left(5.291773 \times 10^{-11} \mathrm{~m}\right)}  \tag{393}\\
B=9.147095+9.147095 j T \tag{394}
\end{gather*}
$$

The time-advanced angular velocity $\omega_{0}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\omega_{0}=2 \pi f_{0}=2 \pi\left(4.808326 \times 10^{15}+4.808326 j \times 10^{15} \mathrm{~Hz}\right)  \tag{395}\\
\omega_{0}=3.02116 \times 10^{16}+3.02116 j \times 10^{16} \mathrm{~Hz} \tag{396}
\end{gather*}
$$

The time-advanced velocity $v_{x}$ of the electron $e^{-}$is,

$$
\begin{gather*}
v_{x}=r \omega_{0}=\left(5.291773 \times 10^{-11} \mathrm{~m}\right)\left(3.02116 \times 10^{16}+3.02116 j \times 10^{16} \mathrm{~Hz}\right)  \tag{397}\\
v_{x}=1.598729 \times 10^{6}+1.598729 j \times 10^{6} \mathrm{~m} / \mathrm{sec} \tag{398}
\end{gather*}
$$

The maximum time-future velocity $v_{x \max }$ within the Earth's gravity well is,

$$
\begin{gather*}
v_{x \max }=\sqrt{-\frac{2 G M_{E}}{y_{0}}}=\sqrt{-\frac{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}}  \tag{399}\\
v_{x \max }=1.11846 j \times 10^{4} \mathrm{~m} / \mathrm{sec} \ll v_{x} \tag{400}
\end{gather*}
$$

The time-advanced velocity $v_{x}$ of the electron $e^{-}$far exceeds coupling to Earth's gravity well!
The maximum time-future velocity $v_{x \max }$ within the Sun's gravity well is,

$$
\begin{gather*}
v_{x \max }=\sqrt{-\frac{2 G M_{S U N}}{y_{0}}}=\sqrt{-\frac{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(1.98892 \times 10^{30} \mathrm{~kg}\right)}{\left(6.96 \times 10^{8} \mathrm{~m}\right)}}  \tag{401}\\
v_{x \max }=6.17542 \mathrm{j} \times 10^{5} \mathrm{~m} / \mathrm{sec} \ll v_{x} \tag{402}
\end{gather*}
$$

The time-advanced velocity $v_{x}$ of the electron $e^{-}$far exceeds coupling to Sun's gravity well!

The time-advanced magnetic force $F_{v}$ of the electron $e^{-}$directed upon the nucleus is,

$$
\begin{gather*}
F_{v}=e^{-} v_{x} B  \tag{403}\\
F_{v}=\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(1.598729 \times 10^{6}+1.598729 \mathrm{j} \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)(9.147095+9.147095 \mathrm{j} T) \\
F_{v}=4.685962 \mathrm{j} \times 10^{-12} \mathrm{~N} \tag{404}
\end{gather*}
$$

The direction of electron motion $v_{x}$ is time-advanced or rotated into the future such that the magnetic force $F_{v}$ is always an attractive force between the electron and the nucleus.

The IMAGINARY fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(1.598729 \times 10^{6}+1.598729 \mathrm{j} \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{405}\\
\Delta M_{e}=2.590585 \mathrm{j} \times 10^{-35} \mathrm{~kg} \tag{406}
\end{gather*}
$$

The special relativistic mass $M_{v}$ of the electron $e^{-}$is,

$$
\begin{gather*}
M_{v}=M_{e}+\Delta M_{e}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right)+\left(2.590585 \mathrm{j} \times 10^{-35} \mathrm{~kg}\right)  \tag{407}\\
M_{v}=9.1093897 \times 10^{-31}+2.590585 \mathrm{j} \times 10^{-35} \mathrm{~kg} \tag{408}
\end{gather*}
$$

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}{ }^{2}}{2 G M_{E}}=} \frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(1.598729 \times 10^{6}+1.598729 \mathrm{j} \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{409}\\
y_{1}=3.819579 \times 10^{-3}-156.0822 \mathrm{j} \mathrm{~m} \tag{410}
\end{gather*}
$$

So, the equivalent IMAGINARY displacement $\Delta y$ is shown to be non-gravitational within the Earth’s gravity well or,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=6.3781 \times 10^{-6}+156.0822 m \tag{411}
\end{equation*}
$$



FIGURE 32. The time-future Bohr model of the Hydrogen atom.
Example 11. In the time-future Bohr model of the Hydrogen atom, the real magnetic field created by an electron circulating at relativistic speeds is being cancelled by an externally applied magnetic field $B_{E X T}$. The electron reacts by rotating its velocity into the imaginary axis as shown above. As a consequence of this complex velocity $j v_{x}$, a complex magnetic field $j B$ emerges.

So, given,
Direction of time $\theta=90^{\circ}$
Frequency of orbit $f=6.8 \times 10^{15} \mathrm{~Hz}$
Permeability of free space $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Classic $1^{\text {st }}$ Bohr orbital radius $r=5.291773 \times 10^{-11} \mathrm{~m}$
Rest mass of an electron $M_{e}=9.1093897 \times 10^{-31} \mathrm{~kg}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mean radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$
Mean radius of surface of Sun $y_{0}=6.96 \times 10^{8} \mathrm{~m}$
Mass of the Sun $M_{\text {SUN }}=1.98892 \times 10^{30} \mathrm{~kg}$

The time-future frequency of orbit $f_{0}$, where $\theta=90^{\circ}$ is,

$$
\begin{equation*}
f_{0}=f e^{j \theta}=\left(6.8 \times 10^{15} \mathrm{~Hz}\right) e^{j 90^{\circ}}=6.8 j \times 10^{15} \mathrm{~Hz} \tag{412}
\end{equation*}
$$

The time-future Amperian Current $i$ is,

$$
\begin{equation*}
i=e^{-} f_{0}=\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(6.8 j \times 10^{15} \mathrm{~Hz}\right)=1.089481 j \times 10^{-3} \mathrm{Amps} \tag{413}
\end{equation*}
$$

The time-future magnetic field $B$ at the center axis of the orbit $z=0 m$ is,

$$
\begin{gather*}
B=\mu_{0} \frac{r^{2} i}{2\left(r^{2}+z^{2}\right)^{3 / 2}}=\mu_{0} \frac{r^{2} e^{-} f_{0}}{2\left(r^{2}+z^{2}\right)^{3 / 2}}  \tag{414}\\
B=\mu_{0} \frac{i}{2 r}=\left(4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right) \frac{\left(1.089481 j \times 10^{-3} \mathrm{Amps}\right)}{2\left(5.291773 \times 10^{-11} \mathrm{~m}\right)}=12.935946 \mathrm{j} T \tag{415}
\end{gather*}
$$

The time-future angular velocity $\omega_{0}$ of the electron $e^{-}$is,

$$
\begin{equation*}
\omega_{0}=2 \pi f_{0}=2 \pi\left(6.8 j \times 10^{15} \mathrm{~Hz}\right)=4.272566 j \times 10^{16} \mathrm{~Hz} \tag{416}
\end{equation*}
$$

The time-future velocity $v_{x}$ of the electron $e^{-}$is,

$$
\begin{equation*}
v_{x}=r \omega_{0}=\left(5.291773 \times 10^{-11} \mathrm{~m}\right)\left(4.272566 j \times 10^{16} \mathrm{~Hz}\right)=2.260945 \mathrm{j} \times 10^{6} \mathrm{~m} / \mathrm{sec} \tag{417}
\end{equation*}
$$

The maximum time-future velocity $v_{x \max }$ within the Earth's gravity well is,

$$
\begin{gather*}
v_{x \max }=\sqrt{-\frac{2 G M_{E}}{y_{0}}}=\sqrt{-\frac{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}}  \tag{418}\\
v_{x \max }=1.11846 \mathrm{j} \times 10^{4} \mathrm{~m} / \mathrm{sec} \ll v_{x} \tag{419}
\end{gather*}
$$

The time-future velocity $v_{x}$ of the electron $e^{-}$far exceeds coupling to Earth's gravity well.

The maximum time-future velocity $v_{x \max }$ within the Sun's gravity well is,

$$
\begin{gather*}
v_{x \max }=\sqrt{-\frac{2 G M_{S U N}}{y_{0}}}=\sqrt{-\frac{2\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(1.98892 \times 10^{30} \mathrm{~kg}\right)}{\left(6.96 \times 10^{8} \mathrm{~m}\right)}}  \tag{420}\\
v_{x \max }=6.17542 \mathrm{j} \times 10^{5} \mathrm{~m} / \mathrm{sec} \ll v_{x} \tag{421}
\end{gather*}
$$

The time-advanced velocity $v_{x}$ of the electron $e^{-}$far exceeds coupling to Sun's gravity well!

The magnetic force $F_{v}$ of the electron $e^{-}$directed upon the nucleus is,

$$
\begin{gather*}
F_{v}=e^{-} v_{x} B=\left(1.602177 \times 10^{-19} \mathrm{C}\right)\left(2.260945 j \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)(12.935946 j T)  \tag{422}\\
F_{v}=-4.685962 \times 10^{-12} \mathrm{~N} \tag{423}
\end{gather*}
$$

The direction of electron motion $v_{x}$ is time-future or rotated into the future such that the magnetic force $F_{v}$ is always an attractive force between the electron and the nucleus.

The NEGATIVE fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{equation*}
\Delta M_{e}=M_{e} \frac{v_{x}{ }^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(2.260945 j \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}=-2.590585 \times 10^{-35} \mathrm{~kg} \tag{424}
\end{equation*}
$$

The decreased special relativistic mass $M_{v}$ of the electron $e^{-}$is,

$$
\begin{equation*}
M_{v}=M_{e}+\Delta M_{e}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right)+\left(-2.590585 \times 10^{-35} \mathrm{~kg}\right)=9.109131 \times 10^{-31} \mathrm{~kg} \tag{425}
\end{equation*}
$$

Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(2.260945 \mathrm{j} \times 10^{6} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{426}\\
y_{1}=-156.0860 \mathrm{~m} \tag{427}
\end{gather*}
$$

So, the equivalent NEGATIVE displacement $\Delta y$ is antigravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=6.378256 \times 10^{6} \mathrm{~m} \tag{428}
\end{equation*}
$$

## COMPLEX ELECTRON DRIFT VELOCITY



FIGURE 33. The complete complex electron drift velocity model in a copper wire.
If a copper wire is connected to a battery, an electric field $E$ will be set up at every point within the wire. This field $E$ will act on electrons and will give them a resultant motion. An electric current $I$ is established if a net charge $q_{C u}$ passes through any cross sectional area $A$ of the conductor in time $t$. The electric field that acts on the electrons doesn't produce a net acceleration because the electrons keep colliding with the atoms that make up the conductor. The electrons, therefore, move at an average drift velocity $v_{x}$. If the electron drift velocity $j v_{x}$ is timefuture, the associated electric field $j E$ and magnetic field $j B$ are also time-future. The complete complex electron drift velocity model includes the following characteristic equations shown below. These equations contain the real and imaginary components of a moving electron that is rotated about the temporal axis as a complex particle. The rotation is given as $0^{\circ} \leq \theta \leq 90^{\circ}$, where the real axis is $\theta=0^{\circ}$ and the imaginary time-future axis is $\theta=90^{\circ}$. The complex number uses the Euler's identity $e^{j \theta}$, which functions as a temporal rotation operator.

So, given,
Direction of time $\theta$
Current flowing through a conductor $I$
Fundamental charge of an electron $e^{-}$
Radius of copper wire $r$
Density of conductor material $\left(20^{\circ} \mathrm{C}\right) D_{\text {atom }}$
Number of conduction electrons per atom of conductor $k_{\text {atom }}$
Avogadro's Number $N_{0}$
Atomic weight of conductor material $W_{\text {atom }}$
Segment length of conductor $x$
Speed of light $c$
Rest mass of an electron $M_{e}$
Radius of surface of Earth $y_{0}$
Gravitational constant $G$
Mass of the Earth $M_{E}$
Resistivity of conductor material $\left(20^{\circ} \mathrm{C}\right) \rho_{\text {atom }}$
The complex current $I_{0}$ flowing through a conductor, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=I \cos \theta+j I \sin \theta \tag{429}
\end{equation*}
$$

The complex current density $J_{0}$ is,

$$
\begin{equation*}
J_{0}=\frac{I_{0}}{A}=\frac{I_{0}}{\pi r^{2}} \tag{430}
\end{equation*}
$$

The volume $v$ of a segment of a conductor is,

$$
\begin{equation*}
V=A x=\pi r^{2} x \tag{431}
\end{equation*}
$$

The quantity of conduction electrons $n_{\text {atom }}$ in a volume of conductor is,

$$
\begin{equation*}
n_{\text {atom }}=\frac{D_{\text {atom }} N_{0} k_{\text {atom }}}{W_{\text {atom }}} \tag{432}
\end{equation*}
$$

The net charge $q_{\text {atom }}$ in a volume of a conductor is,

$$
\begin{equation*}
q_{\text {atom }}=n_{\text {atom }} \vee e^{-} \tag{433}
\end{equation*}
$$

The complex velocity $v_{x}$ is,

$$
\begin{equation*}
v_{x}=\frac{x}{t} \tag{434}
\end{equation*}
$$

The complex current $I_{0}$ flowing through a conductor is,

$$
\begin{equation*}
I_{0}=\frac{q_{\text {atom }}}{t}=\frac{n_{\text {atom }} v e^{-}}{x / v_{x}}=\pi r^{2} n_{\text {atom }} e^{-} v_{x} \tag{435}
\end{equation*}
$$

So, the complex drift velocity $v_{x}$ of an electron moving through a conductor is,

$$
\begin{equation*}
v_{x}=\frac{I_{0}}{\pi r^{2} n_{\text {atom }} e^{-}}=\frac{J_{0}}{n_{\text {atom }} e^{-}} \tag{436}
\end{equation*}
$$

The fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{equation*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}} \tag{437}
\end{equation*}
$$

Applying the new Principle of Equivalence Theorem,

$$
\begin{equation*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=M_{e} \frac{\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}{c^{2}}=G M_{E} M_{e} \frac{\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)}{c^{2}} \tag{438}
\end{equation*}
$$

$$
\begin{gather*}
\frac{v_{x}^{2}}{2}=g_{y_{1}} y_{1}-g_{y_{0}} y_{0}=G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)  \tag{439}\\
v_{x}=\sqrt{2\left(g_{y_{1}} y_{1}-g_{y_{0}} y_{0}\right)}=\sqrt{2 G M_{E}\left(\frac{1}{y_{1}}-\frac{1}{y_{0}}\right)} \tag{440}
\end{gather*}
$$

The equivalent displacement to position $y_{1}$ of an electron moving at a velocity $v_{x}$ within Earth's gravity well $g_{Y}$ where $0<y_{1} \leq \infty$ or $-1 \leq \frac{y_{0} v_{x}^{2}}{2 G M_{E}}$ is,

$$
\begin{gather*}
y_{1}=\frac{1}{g_{y_{1}}}\left(g_{y_{0}} y_{0}+\frac{v_{x}^{2}}{2}\right)  \tag{441}\\
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}} \tag{442}
\end{gather*}
$$

The resistivity $\rho_{\text {Atom }}$ of a conductor is given as,

$$
\begin{equation*}
\rho_{\text {Atom }}=\frac{E}{J}=\frac{V / x}{I / A}=\frac{V / x}{I / \pi r^{2}} \tag{443}
\end{equation*}
$$

The resistance $R$ of a segment of conductor is,

$$
\begin{equation*}
R=\frac{V}{I}=\rho_{\text {Atom }} \frac{x}{A}=\rho_{\text {Atom }} \frac{x}{\pi r^{2}} \tag{444}
\end{equation*}
$$



FIGURE 34. The time-forward electron drift velocity in a copper wire.

Example 12. A time-forward electric current $I$ is established if a net charge $q_{C u}$ passes through any cross sectional area $A$ of the conductor in time-forward $t$. The electrons move at an average time-forward drift velocity $v_{x}$.

So, given,
Direction of time $\theta=0^{\circ}$
Current through copper wire $I=10.0 \mathrm{Amps}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Radius of $10 A W G$ copper wire $r=1.294 \times 10^{-3} \mathrm{~m}$
Density of copper conductor $\left(20^{\circ} \mathrm{C}\right) D_{C u}=8.92 \times 10^{6} \mathrm{gm} / \mathrm{m}^{3}$
Number of conduction electrons per atom of copper $k_{C u}=1$ electron/atom
Avogadro's Number $N_{0}=6.0221367 \times 10^{23}$ atoms $/ \mathrm{mole}$
Atomic weight of copper conductor $W_{C u}=63.546 \mathrm{gm} / \mathrm{mole}$
Segment length $x=1 \mathrm{~m}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Rest mass of an electron $M_{e}=9.1093897 \times 10^{-31} \mathrm{~kg}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mean Radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$
Resistivity of copper conductor $\left(20^{\circ} \mathrm{C}\right) \rho_{C u}=1.68 \times 10^{-8} \Omega \mathrm{~m}$

The time-forward current $I_{0}$ flowing through a conductor, where $\theta=0^{\circ}$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=(10.0 \mathrm{Amps}) e^{j 0^{\circ}}=10.0 \mathrm{Amps} \tag{445}
\end{equation*}
$$

The time-forward current density $J_{0}$ is,

$$
\begin{equation*}
J_{0}=\frac{I_{0}}{A}=\frac{I_{0}}{\pi r^{2}}=\frac{(10.0 \mathrm{Amps})}{\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}}=1.901 \times 10^{6} \mathrm{Amps} / \mathrm{m}^{2} \tag{446}
\end{equation*}
$$

The volume $v$ of a segment of copper wire is,

$$
\begin{equation*}
V=A x=\pi r^{2} x=\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}(1 \mathrm{~m})=5.261 \times 10^{-6} \mathrm{~m}^{3} \tag{447}
\end{equation*}
$$

The quantity of conduction electrons $n_{C u}$ in a volume of copper wire is,

$$
\begin{gather*}
n_{C u}=\frac{D_{C u} N_{0} k_{C u}}{W_{C u}}  \tag{448}\\
n_{C u}=\frac{\left(8.92 \times 10^{6} \mathrm{gm} / \mathrm{m}^{3}\right)\left(6.0221367 \times 10^{23} \text { atoms } / \text { mole }\right)(1 \text { electron } / \text { atom })}{(63.546 \mathrm{gm} / \mathrm{mole})}  \tag{449}\\
n_{C u}=8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3} \tag{450}
\end{gather*}
$$

The net charge $q_{C u}$ in a volume of copper wire is,

$$
\begin{gather*}
q_{C u}=n_{C u} \mho e^{-}=\left(8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3}\right)\left(5.261 \times 10^{-6} \mathrm{~m}^{3}\right)\left(1.60217733 \times 10^{-19} \mathrm{C}\right)  \tag{451}\\
q_{C u}=7.126 \times 10^{4} \mathrm{C} \tag{452}
\end{gather*}
$$

The time-forward velocity $v_{x}$ is,

$$
\begin{equation*}
v_{x}=\frac{x}{t} \tag{453}
\end{equation*}
$$

The time-forward current $I_{0}$ flowing through a copper wire is,

$$
\begin{equation*}
I_{0}=\frac{q_{C u}}{t}=\frac{n_{C u} v e^{-}}{x / v_{x}}=\pi r^{2} n_{C u} e^{-} v_{x}=\pi r^{2} n_{C u} e^{-} v_{x}=10.0 \mathrm{Amps} \tag{454}
\end{equation*}
$$

So, the time-forward drift velocity $v_{x}$ of an electron moving through a copper wire is,

$$
\begin{align*}
v_{x}=\frac{I_{0}}{\pi r^{2} n_{C u} e^{-}}=\frac{J_{0}}{n_{C u} e^{-}}=\frac{\left(1.901 \times 10^{6} \mathrm{Amps} / \mathrm{m}^{2}\right)}{\left(8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3}\right)\left(1.60217733 \times 10^{-19} \mathrm{C}\right)}  \tag{455}\\
v_{x}=1.403 \times 10^{-4} \mathrm{~m} / \mathrm{sec} \tag{456}
\end{align*}
$$

The POSITIVE fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(1.403 \times 10^{-4} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{457}\\
\Delta M_{e}=9.980 \times 10^{-56} \mathrm{~kg} \tag{458}
\end{gather*}
$$

The POSITIVE fluctuating mass of an electron is almost 25 orders of magnitude below its’ rest mass $M_{e}$.
Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(1.403 \times 10^{-4} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{459}\\
y_{1}=6.3781 \times 10^{6} \mathrm{~m} \tag{460}
\end{gather*}
$$

So, the equivalent POSITIVE displacement $\Delta y$ is gravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=1.8627 \times 10^{-9} \mathrm{~m} \tag{461}
\end{equation*}
$$

Given a time-forward voltage $V$ and a time-forward current $I$, the resistivity $\rho_{C u}$ of copper wire is given as,

$$
\begin{equation*}
\rho_{C u}=\frac{E}{J}=\frac{V / x}{I / A}=\frac{V / x}{I / \pi r^{2}}=1.68 \times 10^{-8} \Omega m \tag{462}
\end{equation*}
$$

The resistance $R$ of a segment of copper wire is,

$$
\begin{equation*}
R=\frac{V}{I}=\rho_{C u} \frac{x}{A}=\left(1.68 \times 10^{-8} \Omega \mathrm{~m}\right) \frac{(1 \mathrm{~m})}{\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}}=3.193 \times 10^{-3} \Omega \tag{463}
\end{equation*}
$$



FIGURE 35. The time-advanced electron drift velocity in a copper wire.
Example 13. A time-advanced electric current $I$ is established if a net charge $q_{C u}$ passes through any cross sectional area $A$ of the conductor in time-advanced $t$. The electrons move at an average time-advanced drift velocity $v_{x}$.

So, given,
Direction of time $\theta=45^{\circ}$
Current flow through copper wire $I=10.0 \mathrm{Amps}$
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Radius of $10 A W G$ copper wire $r=1.294 \times 10^{-3} \mathrm{~m}$
Density of copper conductor $\left(20^{\circ} \mathrm{C}\right) D_{C u}=8.92 \times 10^{6} \mathrm{gm} / \mathrm{m}^{3}$
Number of conduction electrons per atom of copper $k_{C u}=1$ electron/atom
Avogadro's Number $N_{0}=6.0221367 \times 10^{23}$ atoms $/ \mathrm{mole}$
Atomic weight of copper conductor $W_{C u}=63.546 \mathrm{gm} / \mathrm{mole}$
Segment length $x=1 m$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Rest mass of an electron $M_{e}=9.1093897 \times 10^{-31} \mathrm{~kg}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mean Radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$
Resistivity of copper conductor $\left(20^{\circ} \mathrm{C}\right) \quad \rho_{\mathrm{Cu}}=1.68 \times 10^{-8} \Omega \mathrm{~m}$

The time-advanced current $I_{0}$ flowing through a conductor, where $\theta=45^{\circ}$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=(10.0 \mathrm{Amps}) e^{j 45^{\circ}}=7.071+7.071 j \mathrm{Amps} \tag{464}
\end{equation*}
$$

The time-advanced current density $J_{0}$ is,

$$
\begin{equation*}
J_{0}=\frac{I_{0}}{A}=\frac{I_{0}}{\pi r^{2}}=\frac{(7.071+7.071 \mathrm{j} \mathrm{Amps})}{\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}}=1.344 \times 10^{6}+1.344 \mathrm{j} \times 10^{6} \mathrm{Amps} / \mathrm{m}^{2} \tag{465}
\end{equation*}
$$

The volume $v$ of a segment of copper wire is,

$$
\begin{equation*}
v=A x=\pi r^{2} x=\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}(1 \mathrm{~m})=5.261 \times 10^{-6} \mathrm{~m}^{3} \tag{466}
\end{equation*}
$$

The quantity of conduction electrons $n_{C u}$ in a volume of copper wire is,

$$
\begin{gather*}
n_{C u}=\frac{D_{C u} N_{0} k_{C u}}{W_{C u}}  \tag{467}\\
\left.n_{C u}=\frac{\left(8.92 \times 10^{6} \mathrm{gm} / \mathrm{m}^{3}\right)\left(6.0221367 \times 10^{23} \text { atoms } / \mathrm{mole}\right)(1 \text { electron } / \text { atom })}{(63.546 \mathrm{gm} / \mathrm{mole})}\right)  \tag{468}\\
n_{C u}=8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3} \tag{469}
\end{gather*}
$$

The net charge $q_{C u}$ in a volume of a copper wire is,

$$
\begin{gather*}
q_{C u}=n_{C u} v e^{-}=\left(8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3}\right)\left(5.261 \times 10^{-6} \mathrm{~m}^{3}\right)\left(1.60217733 \times 10^{-19} \mathrm{C}\right)  \tag{470}\\
q_{C u}=7.126 \times 10^{4} \mathrm{C} \tag{471}
\end{gather*}
$$

The time-advanced velocity $v_{x}$ is,

$$
\begin{equation*}
v_{x}=\frac{x}{t} \tag{472}
\end{equation*}
$$

The time-advanced current $I_{0}$ flowing through a copper wire is,

$$
\begin{equation*}
I_{0}=\frac{q_{C u}}{t}=\frac{n_{C u} v e^{-}}{x / v_{x}}=\pi r^{2} n_{C u} e^{-} v_{x}=7.071+7.071 \mathrm{j} \mathrm{Amps} \tag{473}
\end{equation*}
$$

So, the time-advanced drift velocity $v_{x}$ of an electron moving through a copper wire is,

$$
\begin{equation*}
v_{x}=\frac{I_{0}}{\pi r^{2} n_{C u} e^{-}}=\frac{J_{0}}{n_{C u} e^{-}}=\frac{\left(1.344 \times 10^{6}+1.344 j \times 10^{6} \mathrm{Amps} / \mathrm{m}^{2}\right)}{\left(8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3}\right)\left(1.60217733 \times 10^{-19} \mathrm{C}\right)} \tag{474}
\end{equation*}
$$

$$
\begin{equation*}
v_{x}=9.923 \times 10^{-5}+9.923 \mathrm{j} \times 10^{-5} \mathrm{~m} / \mathrm{sec} \tag{475}
\end{equation*}
$$

The fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(9.923 \times 10^{-5}+9.923 \mathrm{j} \times 10^{-5} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{476}\\
\Delta M_{e}=9.980 \mathrm{j} \times 10^{-56} \mathrm{~kg} \tag{477}
\end{gather*}
$$

The fluctuating mass of an electron is almost 25 orders of magnitude beyond its' rest mass $M_{e}$ and is imaginary.

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(9.923 \times 10^{-5}+9.923 \mathrm{j} \times 10^{-5} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{478}\\
y_{1}=6.3781 \times 10^{6} \mathrm{~m} \tag{479}
\end{gather*}
$$

So, the equivalent IMAGINARY displacement $\Delta y$ is shown to be non-gravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=1.0041 \mathrm{j} \times 10^{-9} \mathrm{~m} \tag{480}
\end{equation*}
$$

Given a time-advanced voltage $V$ and a time-advanced current $I$, the resistivity $\rho_{C u}$ of copper wire is given as,

$$
\begin{equation*}
\rho_{C u}=\frac{E}{J}=\frac{V / x}{I / A}=\frac{V / x}{I / \pi r^{2}}=1.68 \times 10^{-8} \Omega m \tag{481}
\end{equation*}
$$

The resistance $R$ of a segment of copper wire is,

$$
\begin{equation*}
R=\frac{V}{I}=\rho_{C u} \frac{x}{A}=\left(1.68 \times 10^{-8} \Omega m\right) \frac{(1 \mathrm{~m})}{\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}}=3.193 \times 10^{-3} \Omega \tag{482}
\end{equation*}
$$



FIGURE 36. The time-future electron drift velocity in a copper wire.
Example 14. A time-future electric current $+j I$ is established if a net charge $q_{C u}$ passes through any cross sectional area $A$ of the conductor in time-future $t$. The electrons move at an average time-future drift velocity $+j v_{x}$.

So, given,
Direction of time $\theta=90^{\circ}$
Current flow through copper wire $I=10.0$ Amps
Fundamental charge of an electron $e^{-}=1.602177 \times 10^{-19} \mathrm{C}$
Radius of 10 AWG copper wire $r=1.294 \times 10^{-3} \mathrm{~m}$
Density of copper conductor $\left(20^{\circ} \mathrm{C}\right) D_{C u}=8.92 \times 10^{6} \mathrm{gm} / \mathrm{m}^{3}$
Number of conduction electrons per atom of copper $k_{C u}=1$ electron/atom
Avogadro's Number $N_{0}=6.0221367 \times 10^{23}$ atoms $/$ mole
Atomic weight of copper conductor $W_{C u}=63.546 \mathrm{gm} / \mathrm{mole}$
Segment length $x=1 m$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Rest mass of an electron $M_{e}=9.1093897 \times 10^{-31} \mathrm{~kg}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}$
Mean Radius of surface of Earth $y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$
Resistivity of copper conductor $\left(20^{\circ} \mathrm{C}\right) \quad \rho_{\mathrm{Cu}}=1.68 \times 10^{-8} \Omega \mathrm{~m}$

The time-future current $I_{0}$ flowing through a conductor, where $\theta=90^{\circ}$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=(10.0 \mathrm{Amps}) e^{j 90^{\circ}}=10.0 j \mathrm{Amps} \tag{483}
\end{equation*}
$$

The time-future current density $J_{0}$ is,

$$
\begin{equation*}
J_{0}=\frac{I_{0}}{A}=\frac{I_{0}}{\pi r^{2}}=\frac{(10.0 j \mathrm{Amps})}{\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}}=1.901 \mathrm{j} \times 10^{6} \mathrm{Amps} / \mathrm{m}^{2} \tag{484}
\end{equation*}
$$

The volume $v$ of a segment of copper wire is,

$$
\begin{equation*}
V=A x=\pi r^{2} x=\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}(1 \mathrm{~m})=5.261 \times 10^{-6} \mathrm{~m}^{3} \tag{485}
\end{equation*}
$$

The quantity of conduction electrons $n_{C u}$ in a volume of copper wire is,

$$
\begin{gather*}
n_{C u}=\frac{D_{C u} N_{0} k_{C u}}{W_{C u}}  \tag{486}\\
\left.n_{C u}=\frac{\left(8.92 \times 10^{6} \mathrm{gm} / \mathrm{m}^{3}\right)\left(6.0221367 \times 10^{23} \text { atoms } / \mathrm{mole}\right)(1 \text { electron } / \text { atom })}{(63.546 \mathrm{gm} / \mathrm{mole})}\right)  \tag{487}\\
n_{C u}=8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3} \tag{488}
\end{gather*}
$$

The net charge $q_{C u}$ in a volume of a copper wire is,

$$
\begin{gather*}
q_{C u}=n_{C u} v e^{-}=\left(8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3}\right)\left(5.261 \times 10^{-6} \mathrm{~m}^{3}\right)\left(1.60217733 \times 10^{-19} \mathrm{C}\right)  \tag{489}\\
q_{C u}=7.126 \times 10^{4} \mathrm{C} \tag{490}
\end{gather*}
$$

The time-future velocity $v_{x}$ is,

$$
\begin{equation*}
v_{x}=\frac{x}{t} \tag{491}
\end{equation*}
$$

The time-future current $I_{0}$ flowing through a copper wire is,

$$
\begin{equation*}
I_{0}=\frac{q_{C u}}{t}=\frac{n_{C u} v e^{-}}{x / v_{x}}=\pi r^{2} n_{C u} e^{-} v_{x}=\pi r^{2} n_{C u} e^{-} v_{x}=10.0 j \mathrm{Amps} \tag{492}
\end{equation*}
$$

So, the time-future drift velocity $v_{x}$ of an electron moving through a copper wire is,

$$
\begin{gather*}
v_{x}=\frac{I_{0}}{\pi r^{2} n_{C u} e^{-}}=\frac{J_{0}}{n_{C u} e^{-}}=\frac{\left(1.901 \mathrm{j} \times 10^{6} \mathrm{Amps} / \mathrm{m}^{2}\right)}{\left(8.4533 \times 10^{28} \text { electrons } / \mathrm{m}^{3}\right)\left(1.60217733 \times 10^{-19} \mathrm{C}\right)}  \tag{493}\\
v_{x}=1.403 \mathrm{j} \times 10^{-4} \mathrm{~m} / \mathrm{sec} \tag{494}
\end{gather*}
$$

The NEGATIVE fluctuating mass $\Delta M_{e}$ of the electron $e^{-}$is,

$$
\begin{gather*}
\Delta M_{e}=M_{e} \frac{v_{x}^{2}}{2 c^{2}}=\left(9.1093897 \times 10^{-31} \mathrm{~kg}\right) \frac{\left(1.403 \mathrm{j} \times 10^{-4} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}  \tag{495}\\
\Delta M_{e}=-9.980 \times 10^{-56} \mathrm{~kg} \tag{496}
\end{gather*}
$$

The NEGATIVE fluctuating mass of an electron is almost 25 orders of magnitude below its' rest mass $M_{e}$.

## Applying the new Principle of Equivalence Theorem,

$$
\begin{gather*}
y_{1}=\frac{y_{0}}{1+\frac{y_{0} v_{x}^{2}}{2 G M_{E}}}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(1.403 \mathrm{j} \times 10^{-4} \mathrm{~m} / \mathrm{sec}\right)^{2}}{2\left(6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}}  \tag{497}\\
y_{1}=6.3781 \times 10^{6} \mathrm{~m} \tag{498}
\end{gather*}
$$

So, the equivalent NEGATIVE displacement $\Delta y$ is antigravitational within the Earth's gravity well is,

$$
\begin{equation*}
\Delta y=y_{0}-y_{1}=-9.3132 \times 10^{-10} \mathrm{~m} \tag{499}
\end{equation*}
$$

Given a time-future voltage $V$ and a time-future current $I$, the resistivity $\rho_{C u}$ of copper wire is given as,

$$
\begin{equation*}
\rho_{C u}=\frac{E}{J}=\frac{V / x}{I / A}=\frac{V / x}{I / \pi r^{2}}=1.68 \times 10^{-8} \Omega m \tag{500}
\end{equation*}
$$

The resistance $R$ of a segment of copper wire is,

$$
\begin{equation*}
R=\frac{V}{I}=\rho_{C u} \frac{x}{A}=\left(1.68 \times 10^{-8} \Omega \mathrm{~m}\right) \frac{(1 \mathrm{~m})}{\pi\left(1.294 \times 10^{-3} \mathrm{~m}\right)^{2}}=3.193 \times 10^{-3} \Omega \tag{501}
\end{equation*}
$$

## COMPLEX RESISTOR



FIGURE 37. The complete complex resistor.
Given a complex voltage source $V_{S}$ with a temporal rotation operator $e^{j \theta}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is acting upon the voltage, a complex direct current flows through resistor $R$. A complex voltage $V_{R}$ appears across the resistor. The resulting instantaneous power $P_{R}$ is dissipated or absorbed by the resistor.

So, given,
Direction of time $\theta$
Voltage supply $V$
Resistor $R$
The complex voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=V \cos \theta+j V \sin \theta \tag{502}
\end{equation*}
$$

The complex current $I_{R}$ flowing through resistor $R$ is,

$$
\begin{equation*}
I_{R}=\frac{V_{S}}{R} \tag{503}
\end{equation*}
$$

The complex voltage $V_{R}$ across resistor $R$ is,

$$
\begin{equation*}
V_{R}=I_{R} R=V_{S} \tag{504}
\end{equation*}
$$

The resistance $R$ is,

$$
\begin{equation*}
R=\frac{V_{S}}{I_{R}} \tag{505}
\end{equation*}
$$

The instantaneous power $P_{R}$ dissipated and/or absorbed by the resistor is,

$$
\begin{equation*}
P_{R}=V_{R} I_{R}=I_{R}^{2} R=\frac{V_{R}^{2}}{R} \tag{506}
\end{equation*}
$$

Example 15. Given a time-forward voltage source $V_{S}$ and a known resistor value $R$, compute the time-forward current and power dissipated by the resistor.

So, given,
Direction of time $\theta=0^{\circ}$
Voltage Source $V=10.0$ Volts
Resistor $R=2.5 \Omega$

The time-forward voltage $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 0^{\circ}}=10.0 \text { Volts } \tag{507}
\end{equation*}
$$

The time-forward current $I_{R}$ is,

$$
\begin{equation*}
I_{R}=\frac{V_{S}}{R}=\frac{(10.0 \text { Volts })}{(2.5 \Omega)}=4.0 \mathrm{Amps} \tag{508}
\end{equation*}
$$

The instantaneous power $P_{R}$ dissipated by the resistor is,

$$
\begin{equation*}
P_{R}=\frac{V_{R}^{2}}{R}=\frac{(10.0 \mathrm{Volts})^{2}}{(2.5 \Omega)}=40.0 \mathrm{Watts} \tag{509}
\end{equation*}
$$

Example 16. Given a time-advanced voltage source $V_{S}$ and a known resistor value $R$, compute the time-advanced current and power being dissipated and absorbed by the resistor.

So, given,
Direction of time $\theta=45^{\circ}$
Voltage Source $V=10.0$ Volts
Resistor $R=2.5 \Omega$

The time-advanced voltage $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 45^{\circ}}=7.0711+7.0711 \mathrm{jVolts} \tag{510}
\end{equation*}
$$

The time-advanced current $I_{R}$ is,

$$
\begin{equation*}
I_{R}=\frac{V_{S}}{R}=\frac{(7.0711+7.0711 j \text { Volts })}{(2.5 \Omega)}=2.8284+2.8284 j \mathrm{Amps} \tag{511}
\end{equation*}
$$

The instantaneous power $P_{R}$ dissipated and absorbed of the resistor is,

$$
\begin{equation*}
P_{R}=\frac{V_{R}^{2}}{R}=\frac{(7.0711+7.0711 j \text { Volts })^{2}}{(2.5 \Omega)}=40.0 j \text { Watts } \tag{512}
\end{equation*}
$$

The resistor is dissipating and absorbing an equal amount of heat. The resistor is therefore, temperature neutral or adiabatic.

Example 17. Given a time-future voltage source $V_{S}$ and a known resistor value $R$, compute the time-future current and instantaneous power absorbed by the resistor.

So, given,
Direction of time $\theta=90^{\circ}$
Voltage Source $V=10.0$ Volts
Resistor $R=2.5 \Omega$

The time-future voltage $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 90^{\circ}}=10.0 j \text { Volts } \tag{513}
\end{equation*}
$$

The time-future current $I_{R}$ is,

$$
\begin{equation*}
I_{R}=\frac{V_{S}}{R}=\frac{(10.0 j \text { Volts })}{(2.5 \Omega)}=4.0 j \mathrm{Amps} \tag{514}
\end{equation*}
$$

The instantaneous power $P_{R}$ absorbed by the resistor is,

$$
\begin{equation*}
P_{R}=\frac{V_{R}^{2}}{R}=\frac{(10.0 \mathrm{jVolts})^{2}}{(2.5 \Omega)}=-40.0 \mathrm{Watts} \tag{515}
\end{equation*}
$$

## COMPLEX INDUCTOR



FIGURE 38. The complete complex magnetizing inductor.
Given a complex voltage source $V_{S}$ with a temporal rotation operator $e^{j \theta}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is acting upon the voltage, when switch $S$ closes at $t=0 \mathrm{sec}$, a complex direct current $i_{L}$ flows through resistor $R$ and magnetizes inductor $L$. A complex voltage $v_{R}$ appears across the resistor and a complex voltage $v_{L}$ appears across inductor $L$. The resulting instantaneous power $P_{R}$ is dissipated and/or absorbed by the resistor, the instantaneous power $P_{L}$ stored in the inductor, and the energy $E_{L}$ stored in the inductor.

So, given,
Direction of time $\theta$
Time $t$
Voltage supply $V$
Inductor $L$

## Resistor $R$

The complex voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=V \cos \theta+j V \sin \theta \tag{516}
\end{equation*}
$$

The complex voltage across the resistor $R$ is,

$$
\begin{equation*}
v_{R}(t)=i_{L}(t) R \tag{517}
\end{equation*}
$$

The complex voltage across the inductor $L$ is,

$$
\begin{equation*}
v_{L}(t)=L \frac{d i_{L}}{d t} \tag{518}
\end{equation*}
$$

Letting $t_{0}=0 \mathrm{sec}$, the complex current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ is,

$$
\begin{align*}
& V_{S}=v_{R}(t)+v_{L}(t)=i_{L}(t) R+L \frac{d i_{L}}{d t}  \tag{519}\\
& \frac{V_{S}}{R}=i_{L}(t)+\frac{L}{R} \frac{d i_{L}}{d t}  \tag{520}\\
& i_{L}(t)=\frac{V_{S}}{R}-\frac{L}{R} \frac{d i_{L}}{d t}  \tag{521}\\
& \int_{0}^{i_{L}(t)} \frac{1}{i_{L}(t)-\frac{V_{S}}{R}} d i_{L}=-\frac{R}{L} \int_{t_{0}}^{t} d t  \tag{522}\\
& \left.\ln \left(i_{L}(t)-\frac{V_{S}}{R}\right)\right|_{0} ^{i_{L}(t)}=-\left.\frac{R}{L} t\right|_{t_{0}} ^{t}  \tag{523}\\
& \ln \left(i_{L}(t)-\frac{V_{S}}{R}\right)-\ln \left(-\frac{V_{S}}{R}\right)=\ln \left(\frac{i_{L}(t)-\frac{V_{S}}{R}}{-\frac{V_{S}}{R}}\right)=-\frac{R}{L}\left(t-t_{0}\right)  \tag{524}\\
& \frac{i_{L}(t)-\frac{V_{S}}{R}}{-\frac{V_{S}}{R}}=e^{-\frac{R}{L}\left(t-t_{0}\right)}=e^{-\frac{R}{L} t}  \tag{525}\\
& i_{L}(t)=\frac{V_{S}}{R}-\frac{V_{S}}{R} e^{-\frac{R}{L} t}=\frac{V_{S}}{R}\left(1-e^{-\frac{R}{L} t}\right) \tag{526}
\end{align*}
$$

The instantaneous power $P_{R}$ dissipated and/or absorbed by the resistor $R$ is,

$$
\begin{equation*}
P_{R}(t)=v_{R}(t) i_{L}(t)=R i_{L}^{2}(t)=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)^{2} \tag{527}
\end{equation*}
$$

The instantaneous power $P_{L}$ stored in the inductor $L$ is,

$$
\begin{gather*}
P_{L}(t)=v_{L}(t) i_{L}(t)=L i_{L} \frac{d i_{L}}{d t}=V_{S} i_{L}(t)-R i_{L}^{2}(t)  \tag{528}\\
P_{L}(t)=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)-\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)\left(1-\left(1-e^{-\frac{R}{L} t}\right)\right)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{R}{L} t}-e^{-\frac{2 R}{L} t}\right) \tag{529}
\end{gather*}
$$

Letting $t_{0}=0 \mathrm{sec}$, the energy $E_{L}$ stored in the inductor $L$ is,

$$
\begin{gather*}
E_{L}(t)=\int_{t_{0}}^{t} P_{L} d t=L \int_{t_{0}}^{t} i_{L} \frac{d i_{L}}{d t} d t=L \int_{i\left(t_{0}\right)}^{i(t)} i_{L} d i_{L}=\frac{1}{2} L\left[\left[i_{L}(t)\right]^{2}-\left[i_{L}\left(t_{0}\right)\right]^{2}\right]  \tag{530}\\
E_{L}(t)=\frac{1}{2} L\left[\left[\frac{V_{S}}{R}\left(1-e^{-\frac{R}{L} t}\right)\right]^{2}-\left[\frac{V_{S}}{R}\left(1-e^{-\frac{R}{L} t_{0}}\right)\right]^{2}\right]=\frac{L V_{S}^{2}}{2 R^{2}}\left[\left(1-e^{-\frac{R}{L} t}\right)^{2}-\left(1-e^{-\frac{R}{L} t_{0}}\right)^{2}\right]  \tag{531}\\
E_{L}(t)=\frac{L V_{S}^{2}}{2 R^{2}}\left(1-e^{-\frac{R}{L} t}\right)^{2} \tag{532}
\end{gather*}
$$

Example 18. Given a time-forward voltage source $V_{S}$, a known resistor value $R$ and inductor value $L$, compute the time-forward current and power dissipated by the resistor, and the energy stored in the inductor.

So, given,
Direction of time $\theta=0^{\circ}$
Time 0.0 sec $\leq t \leq 1.0$ sec
Voltage supply $V=10.0$ Volts
Inductor $L=470 \mathrm{mH}$
Resistor $R=2.5 \Omega$

The time-forward voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 0^{\circ}}=10.0 \text { Volts } \tag{533}
\end{equation*}
$$

The time-forward current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
i_{L}(t)=\frac{V_{S}}{R}\left(1-e^{-\frac{R}{L} t}\right)=\frac{(10.0 \text { Volts })}{(2.5 \Omega)}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{534}\\
i_{L}(1.0 \mathrm{sec})=3.980 \mathrm{Amps} \tag{535}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated by the resistor $R$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{(10.0 \text { Volts })^{2}}{(2.5 \Omega)}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})^{t}}}\right)^{2}  \tag{536}\\
P_{R}(1.0 \mathrm{sec})=39.609 \mathrm{Watts} \tag{537}
\end{gather*}
$$

The instantaneous power $P_{L}$ stored in the inductor $L$ at $t=0.130 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{L}(t)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{R}{L} t}-e^{-\frac{2 R}{L} t}\right)=\frac{(10.0 \text { Volts })^{2}}{(2.5 \Omega)}\left(e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}-e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{538}\\
P_{L}(0.130 \mathrm{sec})=10.0 \text { Watts }  \tag{539}\\
P_{L}(1.0 \mathrm{sec})=0.195 \text { Watts } \tag{540}
\end{gather*}
$$

The energy $E_{L}$ stored in the inductor $L$ from $t_{0}=0.0 \mathrm{sec}$ to $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{L}(t)=\frac{L V_{S}^{2}}{2 R^{2}}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{(470 \mathrm{mH})(10.0 \text { Volts })^{2}}{2(2.5 \Omega)^{2}}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)^{2}  \tag{541}\\
E_{L}(1.0 \mathrm{sec})=3.723 \text { Joules } \tag{542}
\end{gather*}
$$

Example 19. Given a time-advanced voltage source $V_{S}$, a known resistor value $R$ and inductor value $L$, compute the time-advanced current and power dissipated and absorbed by the resistor, and the energy stored in the inductor.

So, given,
Direction of time $\theta=45^{\circ}$
Time 0.0 sec $\leq t \leq 1.0$ sec
Voltage supply $V=10.0$ Volts
Inductor $L=470 \mathrm{mH}$
Resistor $R=2.5 \Omega$

The time-advanced voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 45^{\circ}}=7.071+7.071 \mathrm{jVolts} \tag{543}
\end{equation*}
$$

The time-advanced current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
i_{L}(t)=\frac{V_{S}}{R}\left(1-e^{-\frac{R}{L} t}\right)=\frac{(7.071+7.071 j \text { Volts })}{(2.5 \Omega)}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{544}\\
i_{L}(1.0 \mathrm{sec})=2.815+2.815 \mathrm{j} \mathrm{Amps} \tag{545}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and absorbed by the resistor $R$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{(7.071+7.071 j \text { Volts })^{2}}{(2.5 \Omega)}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)^{2}  \tag{546}\\
P_{R}(1.0 \mathrm{sec})=39.609 \mathrm{j} \text { Watts } \tag{547}
\end{gather*}
$$

The instantaneous power $P_{L}$ stored in the inductor $L$ at $t=0.130 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{L}(t)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{R}{L} t}-e^{-\frac{2 R}{L} t}\right)=\frac{(7.071+7.071 j \text { Volts })^{2}}{(2.5 \Omega)}\left(e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}-e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{548}\\
P_{L}(0.130 \mathrm{sec})=10.0 j \text { Watts }  \tag{549}\\
P_{L}(1.0 \mathrm{sec})=0.195 j \text { Watts } \tag{550}
\end{gather*}
$$

The energy $E_{L}$ stored in the inductor $L$ from $t_{0}=0.0 \mathrm{sec}$ to $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{L}(t)=\frac{L V_{S}^{2}}{2 R^{2}}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{(470 \mathrm{mH})(7.071+7.071 j \text { Volts })^{2}}{2(2.5 \Omega)^{2}}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)^{2}  \tag{551}\\
E_{L}(1.0 \mathrm{sec})=3.723 j \text { Joules } \tag{552}
\end{gather*}
$$

Example 20. Given a time-future voltage source $V_{S}$, a known resistor value $R$ and inductor value $L$, compute the time-future current and power absorbed by the resistor, and the negative energy stored in the inductor.

So, given,
Direction of time $\theta=90^{\circ}$
Time 0.0 sec $\leq t \leq 1.0$ sec
Voltage supply $V=10.0$ Volts
Inductor $L=470 \mathrm{mH}$
Resistor $R=2.5 \Omega$

The time-future voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 90^{\circ}}=10.0 j \text { Volts } \tag{553}
\end{equation*}
$$

The time-future $i_{L}$ flowing through the resistor $R$ and the inductor $L$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
i_{L}(t)=\frac{V_{S}}{R}\left(1-e^{-\frac{R}{L} t}\right)=\frac{(10.0 j \text { Volts })}{(2.5 \Omega)}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{554}\\
i_{L}(1.0 \mathrm{sec})=3.980 j \text { Amps } \tag{555}
\end{gather*}
$$

The instantaneous power $P_{R}$ absorbed by the resistor $R$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{(10.0 j \text { Volts })^{2}}{(2.5 \Omega)}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)^{2}  \tag{556}\\
P_{R}(1.0 \mathrm{sec})=-39.609 \mathrm{Watts} \tag{557}
\end{gather*}
$$

The instantaneous power $P_{L}$ stored in the inductor $L$ at $t=0.130 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{L}(t)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{R}{L} t}-e^{-\frac{2 R}{L} t}\right)=\frac{(10.0 j \text { Volts })^{2}}{(2.5 \Omega)}\left(e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})^{t}}}-e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{558}\\
P_{L}(0.130 \mathrm{sec})=-10.0 \mathrm{Watts}  \tag{559}\\
P_{L}(1.0 \mathrm{sec})=-0.195 \mathrm{Watts} \tag{560}
\end{gather*}
$$

The energy $E_{L}$ stored in the inductor $L$ from $t_{0}=0.0 \mathrm{sec}$ to $t=1.0 \mathrm{sec}$ is,

$$
\begin{equation*}
E_{L}(t)=\frac{L V_{S}^{2}}{2 R^{2}}\left(1-e^{-\frac{R}{L} t}\right)^{2}=\frac{(470 \mathrm{mH})(10.0 j \text { Volts })^{2}}{2(2.5 \Omega)^{2}}\left(1-e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)^{2} \tag{561}
\end{equation*}
$$

$$
\begin{equation*}
E_{L}(1.0 \mathrm{sec})=-3.723 \text { Joules } \tag{562}
\end{equation*}
$$



FIGURE 39. The complete complex demagnetizing inductor.
Given energy $E_{L}$ stored in inductor $L$ with a temporal rotation operator $e^{j \theta}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is acting upon the voltage, when switch $S$ closes at $t=0 \mathrm{sec}$, a complex direct current $i_{L}$ flows through resistor $R$. The inductor $L$ demagnetizes into the resistor. A complex voltage $v_{R}$ appears across the resistor $R$. The resulting instantaneous power $P_{R}$ and energy $E_{R}$ are dissipated and/or absorbed by the resistor.

So, given,
Direction of time $\theta$
Time $t$
Initial current through inductor $I$
Inductor $L$
Resistor $R$

The complex current $I_{0}$ through the inductor $L$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=I \cos \theta+j I \sin \theta \tag{563}
\end{equation*}
$$

At $t=0 \mathrm{sec}$, the voltage $V_{0}$ across the resistor $R$ is,

$$
\begin{equation*}
V_{0}=I_{0} R \tag{564}
\end{equation*}
$$

The complex voltage $v_{R}$ across the resistor $R$ is,

$$
\begin{equation*}
v_{R}(t)=R i_{L}(t) \tag{565}
\end{equation*}
$$

The complex voltage $v_{L}$ across the inductor $L$ is,

$$
\begin{equation*}
v_{L}(t)=-L \frac{d i_{L}}{d t} \tag{566}
\end{equation*}
$$

Letting $t_{0}=0 \mathrm{sec}$, the complex current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ is,

$$
\begin{gather*}
v_{R}(t)=v_{L}(t)  \tag{567}\\
i_{L}(t) R=-L \frac{d i_{L}}{d t}  \tag{568}\\
i_{L}(t)=-\frac{L}{R} \frac{d i_{L}}{d t}  \tag{569}\\
\frac{1}{i_{L}(t)} d i_{L}=-\frac{R}{L} d t  \tag{570}\\
\int_{I_{0}}^{i(t)} \frac{1}{i_{L}(t)} d i_{L}=-\frac{R}{L} \int_{t_{0}}^{t} d t  \tag{571}\\
\left.\ln \left(i_{L}(t)\right)\right|_{I_{0}} ^{i_{L}(t)}=-\left.\frac{R}{L} t\right|_{t_{0}} ^{t}  \tag{572}\\
\ln \left(i_{L}(t)\right)-\ln \left(I_{0}\right)=\ln \left(\frac{i_{L}(t)}{I_{0}}\right)=-\frac{R}{L}\left(t-t_{0}\right)  \tag{573}\\
\frac{i_{L}(t)}{I_{0}}=e^{-\frac{R}{L}\left(t-t_{0}\right)}  \tag{574}\\
i_{L}(t)=I_{0} e^{-\frac{R}{L} t} \tag{575}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and/or absorbed by the resistor $R$ is,

$$
\begin{equation*}
P_{R}(t)=v_{R}(t) i_{L}(t)=R i_{L}^{2}(t)=R I_{0}^{2} e^{-\frac{2 R}{L} t} \tag{576}
\end{equation*}
$$

Letting $t_{0}=0 \mathrm{sec}$, the energy $E_{R}$ dissipated and/or absorbed by the resistor $R$ is,

$$
\begin{gather*}
E_{R}(t)=\int_{t_{0}}^{t} P_{R} d t=R I_{0}^{2} \int_{t_{0}}^{t} e^{-\frac{2 R}{L} t} d t  \tag{577}\\
E_{R}(t)=R I_{0}^{2}\left(\frac{-L}{2 R}\right) e^{-\frac{2 R}{L} t}-R I_{0}^{2}\left(\frac{-L}{2 R}\right) e^{-\frac{2 R}{L} t_{0}}==\frac{L I_{0}^{2}}{2}\left[e^{-\frac{2 R}{L} t_{0}}-e^{-\frac{2 R}{L} t}\right]  \tag{578}\\
E_{R}(t)=\frac{L I_{0}^{2}}{2}\left[1-e^{-\frac{2 R}{L} t}\right] \tag{579}
\end{gather*}
$$

Example 21. Given a time-forward voltage $V$ across inductor $L$, a known resistor value $R$ and inductor value $L$, compute the time-forward current flowing through the resistor, and the power and energy dissipated by the resistor.

So, given,
Direction of time $\theta=0^{\circ}$
Time $0.0 \mathrm{sec} \leq t \leq 1.0 \mathrm{sec}$
Initial current through inductor $I=4.0 \mathrm{Amps}$
Inductor $L=470 \mathrm{mH}$
Resistor $R=2.5 \Omega$

The time-forward current $I_{0}$ through the inductor $L$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=(4.0 \mathrm{Amps}) e^{j 0^{\circ}}=4.0 \mathrm{Amps} \tag{580}
\end{equation*}
$$

The time-forward current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
i_{L}(t)=I_{0} e^{-\frac{R}{L} t}=(4.0 \mathrm{Amps}) e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}  \tag{581}\\
i_{L}(0.0 \mathrm{sec})=4.0 \mathrm{Amps}  \tag{582}\\
i_{L}(1.0 \mathrm{sec})=0.020 \mathrm{Amps} \tag{583}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=R I_{0}^{2} e^{-\frac{2 R}{L} t}=(2.5 \Omega)(4.0 \mathrm{Amps})^{2} e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}  \tag{584}\\
P_{R}(0.0 \mathrm{sec})=40.0 \mathrm{Watts}  \tag{585}\\
P_{R}(1.0 \mathrm{sec})=9.592 \times 10^{-4} \mathrm{Watts} \tag{586}
\end{gather*}
$$

The energy $E_{R}$ dissipated by the resistor $R$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{R}(t)=\frac{L I_{0}^{2}}{2}\left[1-e^{-\frac{2 R}{L} t}\right]=\frac{(470 \mathrm{mH})(4.0 \text { Amps })^{2}}{2}\left(1-e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{587}\\
E_{R}(1.0 \mathrm{sec})=3.760 \text { Joules } \tag{588}
\end{gather*}
$$

Example 22. Given a time-advanced voltage $V$ across inductor $L$, a known resistor value $R$ and inductor value $L$, compute the time-advanced current flowing through the resistor, and the power and energy dissipated and absorbed by the resistor.

So, given,
Direction of time $\theta=45^{\circ}$
Time $0.0 \mathrm{sec} \leq t \leq 1.0 \mathrm{sec}$
Initial current through inductor $I=4.0 \mathrm{Amps}$
Inductor $L=470 \mathrm{mH}$
Resistor $R=2.5 \Omega$

The time-advanced current $I_{0}$ through the inductor $L$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=(4.0 \mathrm{Amps}) e^{j 45^{\circ}}=2.828+2.828 j \mathrm{Amps} \tag{589}
\end{equation*}
$$

The time-advanced current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
i_{L}(t)=I_{0} e^{-\frac{R}{L} t}=(2.828+2.828 j \mathrm{Amps}) e^{-\frac{(2.5 \Omega)}{(470 m H)} t}  \tag{590}\\
i_{L}(0.0 \mathrm{sec})=2.828+2.828 \mathrm{j} \mathrm{Amps}  \tag{591}\\
i_{L}(1.0 \mathrm{sec})=0.014+0.014 \mathrm{j} \mathrm{Amps} \tag{592}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=R I_{0}{ }^{2} e^{-\frac{2 R}{L} t}=(2.5 \Omega)(2.828+2.828 j \text { Amps })^{2} e^{-\frac{2(2.5 \Omega)}{(470 m H)} t}  \tag{593}\\
P_{R}(0.0 \mathrm{sec})=40.0 j \text { Watts }  \tag{594}\\
P_{R}(1.0 \mathrm{sec})=9.592 j \times 10^{-4} \mathrm{Watts} \tag{595}
\end{gather*}
$$

The energy $E_{R}$ dissipated and absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{R}(t)=\frac{L I_{0}^{2}}{2}\left[1-e^{-\frac{2 R}{L} t}\right]=\frac{(470 \mathrm{mH})(2.828+2.828 j \text { Amps })^{2}}{2}\left(1-e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{596}\\
E_{R}(1.0 \mathrm{sec})=3.760 j \text { Joules } \tag{597}
\end{gather*}
$$

Example 23. Given a time-future voltage $V$ across inductor $L$, a known resistor value $R$ and inductor value $L$, compute the time-future current flowing through the resistor, and the power and energy absorbed by the resistor.

So, given,
Direction of time $\theta=90^{\circ}$
Time $0.0 \mathrm{sec} \leq t \leq 1.0 \mathrm{sec}$
Initial current through inductor $I=4.0 \mathrm{Amps}$
Inductor $L=470 \mathrm{mH}$
Resistor $R=2.5 \Omega$

The time-future current $I_{0}$ through the inductor $L$ is,

$$
\begin{equation*}
I_{0}=I e^{j \theta}=(4.0 \mathrm{Amps}) e^{j 90^{\circ}}=4.0 j \mathrm{Amps} \tag{598}
\end{equation*}
$$

The time-future current $i_{L}$ flowing through the resistor $R$ and the inductor $L$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
i_{L}(t)=I_{0} e^{-\frac{R}{L} t}=(4.0 j \mathrm{Amps}) e^{-\frac{(2.5 \Omega)}{(470 \mathrm{mH})} t}  \tag{599}\\
i_{L}(0.0 \mathrm{sec})=4.0 j \mathrm{Amps}  \tag{600}\\
i_{L}(1.0 \mathrm{sec})=0.020 j \mathrm{Amps} \tag{601}
\end{gather*}
$$

The instantaneous power $P_{R}$ absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=R I_{0}^{2} e^{-\frac{2 R}{L} t}=(2.5 \Omega)(4.0 j \mathrm{Amps})^{2} e^{-\frac{2(2.5 \Omega)}{(470 m H)} t}  \tag{602}\\
P_{R}(0.0 \mathrm{sec})=-40.0 \mathrm{Watts}  \tag{603}\\
P_{R}(1.0 \mathrm{sec})=-9.592 \times 10^{-4} \mathrm{Watts} \tag{604}
\end{gather*}
$$

The energy $E_{R}$ absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{R}(t)=\frac{L I_{0}^{2}}{2}\left[1-e^{-\frac{2 R}{L} t}\right]=\frac{(470 \mathrm{mH})(4.0 j \mathrm{Amps})^{2}}{2}\left(1-e^{-\frac{2(2.5 \Omega)}{(470 \mathrm{mH})} t}\right)  \tag{605}\\
E_{R}(1.0 \mathrm{sec})=-3.760 \text { Joules } \tag{606}
\end{gather*}
$$

## COMPLEX CAPACITOR



FIGURE 40. The complete complex charging capacitor.
Given a complex voltage source $V_{S}$ with a temporal rotation operator $e^{j \theta}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is acting upon the voltage, when switch $S$ closes at $t=0 \mathrm{sec}$, a complex direct current flows through resistor $R$ and charges capacitor $C$. A complex voltage $v_{R}$ appears across the resistor and a complex voltage $v_{C}$ appears across capacitor $C$. The resulting power $P_{R}$ is dissipated and/or absorbed by the resistor and the energy $E_{C}$ is stored in the capacitor.

So, given,
Direction of time $\theta$
Time $t$
Voltage supply $V$
Capacitor C
Resistor $R$
The complex voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=V \cos \theta+j V \sin \theta \tag{607}
\end{equation*}
$$

The complex voltage $v_{R}$ across the resistor $R$ is,

$$
\begin{equation*}
v_{R}(t)=i_{C}(t) R \tag{608}
\end{equation*}
$$

The complex current $i_{C}$ through a capacitor $C$ is,

$$
\begin{equation*}
i_{C}(t)=C \frac{d v_{C}}{d t} \tag{609}
\end{equation*}
$$

Letting $t_{0}=0 \mathrm{sec}$, the complex voltage $v_{C}$ across the capacitor $C$ is,

$$
\begin{gather*}
V_{S}=v_{R}(t)+v_{C}(t)=i_{C}(t) R+v_{C}(t)  \tag{610}\\
v_{C}(t)=V_{S}-R C \frac{d v_{C}}{d t}  \tag{611}\\
\frac{1}{v_{C}(t)-V_{S}} d v_{C}=-\frac{1}{R C} d t \tag{612}
\end{gather*}
$$

$$
\begin{gather*}
\int_{0}^{v_{C}(t)} \frac{1}{v_{C}(t)-V_{S}} d v_{C}=-\frac{1}{R C} \int_{t_{0}}^{t} d t  \tag{613}\\
\left.\ln \left(v_{C}(t)-V_{S}\right)\right|_{0} ^{v_{C}(t)}=-\left.\frac{1}{R C} t\right|_{t_{0}} ^{t}  \tag{614}\\
\ln \left(v_{C}(t)-V_{S}\right)-\ln \left(-V_{S}\right)=\ln \left(\frac{v_{C}(t)-V_{S}}{-V_{S}}\right)=-\frac{1}{R C}\left(t-t_{0}\right)  \tag{615}\\
\frac{v_{C}(t)-V_{S}}{-V_{S}}=e^{-\frac{1}{R C}\left(t-t_{0}\right)}=e^{-\frac{1}{R C} t}  \tag{616}\\
v_{C}(t)=V_{S}-V_{S} e^{-\frac{1}{R C} t}=V_{S}\left(1-e^{-\frac{1}{R C} t}\right) \tag{617}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and/or absorbed by the resistor $R$ is,

$$
\begin{equation*}
P_{R}(t)=v_{R}(t) i_{C}(t)=\frac{v_{R}^{2}(t)}{R}=\frac{\left(V_{S}-v_{C}(t)\right)^{2}}{R}=\frac{V_{S}^{2}}{R} e^{-\frac{2}{R C} t} \tag{618}
\end{equation*}
$$

The instantaneous power $P_{C}$ stored in the capacitor $C$ is,

$$
\begin{gather*}
P_{C}(t)=v_{C}(t) i_{C}(t)=C v_{C} \frac{d v_{C}}{d t}=\frac{\left(V_{S} v_{C}(t)-v_{C}^{2}(t)\right)}{R}  \tag{619}\\
P_{C}(t)=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{1}{R C} t}\right)-\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{1}{R C} t}\right)^{2}=\frac{V_{S}^{2}}{R}\left(1-e^{-\frac{1}{R C} t}\right)\left(1-\left(1-e^{-\frac{1}{R C} t}\right)\right)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{1}{R C} t}-e^{-\frac{2}{R C} t}\right) \tag{620}
\end{gather*}
$$

Letting $t_{0}=0$ sec , the energy $E_{C}$ stored in the capacitor $C$ is,

$$
\begin{gather*}
E_{L}(t)=\int_{t_{0}}^{t} P_{L} d t=C \int_{t_{0}}^{t} v_{C} \frac{d v_{C}}{d t} d t=C \int_{v\left(t_{0}\right)}^{v(t)} v_{C} d v_{C}=\frac{1}{2} C\left[\left[v_{C}(t)\right]^{2}-\left[v_{C}\left(t_{0}\right)\right]^{2}\right]  \tag{621}\\
E_{C}(t)=\frac{1}{2} C\left[\left[V_{S}\left(1-e^{-\frac{1}{R C} t}\right)\right]^{2}-\left[V_{S}\left(1-e^{-\frac{1}{R C} t_{0}}\right)\right]^{2}\right]=\frac{C V_{S}^{2}}{2}\left[\left(1-e^{-\frac{1}{R C} t}\right)^{2}-\left(1-e^{-\frac{1}{R C} t_{0}}\right)^{2}\right]  \tag{622}\\
E_{C}(t)=\frac{C V_{S}^{2}}{2}\left(1-e^{-\frac{1}{R C} t}\right)^{2} \tag{623}
\end{gather*}
$$

Example 24. Given a time-forward voltage source $V_{S}$, a known resistor value $R$ and capacitor value $C$, compute the time-forward current and power dissipated by the resistor, and the energy stored in the inductor.

So, given,
Direction of time $\theta=0^{\circ}$

Time $0.0 \mathrm{sec} \leq t \leq 1.0$ sec
Voltage supply $V=10.0$ Volts
Capacitor $C=470 \mu F$
Resistor $R=1.0 \mathrm{k} \Omega$

The time-forward voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 0^{\circ}}=10.0 \text { Volts } \tag{624}
\end{equation*}
$$

The time-forward voltage $v_{C}$ across the capacitor $C$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
v_{C}(t)=V_{S}\left(1-e^{-\frac{1}{R C} t}\right)=(10.0 \text { Volts })\left(1-e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{625}\\
v_{C}(1.0 \mathrm{sec})=8.809 \text { Volts } \tag{626}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{S}^{2}}{R} e^{-\frac{2}{R C} t}=\frac{(10.0 \mathrm{Volts})^{2}}{(1.0 \mathrm{k} \Omega)} e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu \mathrm{~F})} t}  \tag{627}\\
P_{R}(0.0 \mathrm{sec})=0.10 \mathrm{Watts}  \tag{628}\\
P_{R}(1.0 \mathrm{sec})=1.419 \times 10^{-3} \mathrm{Watts} \tag{629}
\end{gather*}
$$

The instantaneous power $P_{C}$ stored in the capacitor $C$ at $t=0.33 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{C}(t)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{1}{R C} t}-e^{-\frac{2}{R C} t}\right)=\frac{(10.0 \text { Volts })^{2}}{(1.0 \mathrm{k} \Omega)}\left(e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}-e^{-\frac{2}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{630}\\
P_{C}(0.33 \mathrm{sec})=0.0245 \text { Watts }  \tag{631}\\
P_{C}(1.0 \mathrm{sec})=0.0105 \text { Watts } \tag{632}
\end{gather*}
$$

The energy $E_{C}$ stored in the capacitor $C$ from $t_{0}=0.0 \mathrm{sec}$ to $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{C}(t)=\frac{C V_{S}^{2}}{2}\left(1-e^{-\frac{1}{R C} t}\right)^{2}=\frac{(470 \mu F)(10.0 \text { Volts })^{2}}{2}\left(1-e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}\right)^{2}  \tag{633}\\
E_{C}(t)=0.0182 \text { Joules } \tag{634}
\end{gather*}
$$

Example 25. Given a time-advanced voltage source $V_{S}$, a known resistor value $R$ and capacitor value $C$, compute the time-advanced current and power dissipated and absorbed by the resistor, and the energy stored in the inductor.

So, given,

Direction of time $\theta=45^{\circ}$
Time 0.0 sec $\leq t \leq 1.0$ sec
Voltage supply $V=10.0$ Volts
Capacitor $C=470 \mu F$
Resistor $R=1.0 \mathrm{k} \Omega$

The time-advanced voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 45^{\circ}}=7.071+7.071 j \text { Volts } \tag{635}
\end{equation*}
$$

The time-advanced voltage $v_{C}$ across the capacitor $C$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
v_{C}(t)=V_{S}\left(1-e^{-\frac{1}{R C} t}\right)=(7.071+7.071 j \text { Volts })\left(1-e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{636}\\
v_{C}(1.0 \mathrm{sec})=6.229+6.229 j \text { Volts } \tag{637}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{S}^{2}}{R} e^{-\frac{2}{R C} t}=\frac{(7.071+7.071 j \text { Volts })^{2}}{(1.0 \mathrm{k} \Omega)} e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu \mathrm{~F})} t}{ }^{t}  \tag{638}\\
P_{R}(0.0 \mathrm{sec})=0.10 j \text { Watts }  \tag{639}\\
P_{R}(1.0 \mathrm{sec})=1.419 \mathrm{j} \times 10^{-3} \mathrm{Watts} \tag{640}
\end{gather*}
$$

The instantaneous power $P_{C}$ stored in the capacitor $C$ at $t=0.33 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{align*}
P_{C}(t)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{1}{R C} t}-e^{-\frac{2}{R C} t}\right)= & \frac{(7.071+7.071 j \text { Volts })^{2}}{(1.0 k \Omega)}\left(e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}-e^{-\frac{2}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{641}\\
& P_{C}(0.33 \mathrm{sec})=0.0245 j \text { Watts }  \tag{642}\\
& P_{C}(1.0 \mathrm{sec})=0.0105 \text { Watts } \tag{643}
\end{align*}
$$

The energy $E_{C}$ stored in the capacitor $C$ from $t_{0}=0.0 \mathrm{sec}$ to $t=1.0 \mathrm{sec}$ is,

$$
\begin{align*}
E_{C}(t)=\frac{C V_{S}^{2}}{2}\left(1-e^{-\frac{1}{R C} t}\right)^{2}= & \frac{(470 \mu F)(7.071+7.071 j \text { Volts })^{2}}{2}\left(1-e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}\right)^{2}  \tag{644}\\
& E_{C}(t)=0.0182 j \text { Joules } \tag{645}
\end{align*}
$$

Example 26. Given a time-future voltage source $V_{S}$, a known resistor value $R$ and capacitor value $C$, compute the time-future current and power absorbed by the resistor, and the energy stored in the inductor.

So, given,
Direction of time $\theta=90^{\circ}$
Time 0.0 sec $\leq t \leq 1.0$ sec
Voltage supply $V=10.0$ Volts
Capacitor $C=470 \mu F$
Resistor $R=1.0 \mathrm{k} \Omega$

The time-future voltage supply $V_{S}$ is,

$$
\begin{equation*}
V_{S}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 90^{\circ}}=10.0 j \text { Volts } \tag{646}
\end{equation*}
$$

The time-future voltage $v_{C}$ across the capacitor $C$ at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
v_{C}(t)=V_{S}\left(1-e^{-\frac{1}{R C} t}\right)=(10.0 j \text { Volts })\left(1-e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{647}\\
v_{C}(1.0 \mathrm{sec})=8.809 j \text { Volts } \tag{648}
\end{gather*}
$$

The instantaneous power $P_{R}$ absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{S}^{2}}{R} e^{-\frac{2}{R C} t}=\frac{(10.0 j \text { Volts })^{2}}{(1.0 \mathrm{k} \Omega)} e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu \mathrm{~F})} t}  \tag{649}\\
P_{R}(0.0 \mathrm{sec})=-0.10 \mathrm{Watts}  \tag{650}\\
P_{R}(1.0 \mathrm{sec})=-1.419 \times 10^{-3} \mathrm{Watts} \tag{651}
\end{gather*}
$$

The instantaneous power $P_{C}$ stored in the capacitor $C$ at $t=0.33 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{C}(t)=\frac{V_{S}^{2}}{R}\left(e^{-\frac{1}{R C} t}-e^{-\frac{2}{R C} t}\right)=\frac{(10.0 j \text { Volts })^{2}}{(1.0 \mathrm{k} \Omega)}\left(e^{-\frac{1}{(1.0 k \Omega)(470 \mu \mathrm{~F})} t}-e^{-\frac{2}{(1.0 k \Omega)(470 \mu \mathrm{~F})} t}\right)  \tag{652}\\
P_{C}(0.33 \mathrm{sec})=-0.0245 \text { Watts }  \tag{653}\\
P_{C}(1.0 \mathrm{sec})=-0.0105 \text { Watts } \tag{654}
\end{gather*}
$$

The energy $E_{C}$ stored in the capacitor $C$ from $t_{0}=0.0 \mathrm{sec}$ to $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{C}(t)=\frac{C V_{S}^{2}}{2}\left(1-e^{-\frac{1}{R C} t}\right)^{2}=\frac{(470 \mu F)(10.0 j \text { Volts })^{2}}{2}\left(1-e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}\right)^{2}  \tag{655}\\
E_{C}(t)=-0.0182 \text { Joules } \tag{656}
\end{gather*}
$$



FIGURE 41. The complete complex discharging capacitor.
Given energy $E_{C}$ stored in capacitor $C$ with a temporal rotation operator $e^{j \theta}$, where $0^{\circ} \leq \theta \leq 90^{\circ}$ is acting upon the voltage, when switch $S$ closes at $t=0 \mathrm{sec}$, a complex direct current $i_{C}$ flows through resistor $R$. The capacitor $C$ discharges into the resistor. A complex voltage $v_{R}$ appears across the resistor $R$. The resulting instantaneous power $P_{R}$ and energy $E_{R}$ are dissipated and/or absorbed by the resistor.

So, given,
Direction of time $\theta$
Time $t$
Initial voltage across capacitor $V$
Capacitor C
Resistor $R$
At $t=0 \mathrm{sec}$, the voltage $V_{0}$ across the resistor $R$ is,

$$
\begin{equation*}
V_{0}=I_{0} R \tag{657}
\end{equation*}
$$

The complex voltage $v_{R}$ across the resistor $R$ is,

$$
\begin{equation*}
v_{R}(t)=R i_{C}(t) \tag{658}
\end{equation*}
$$

The complex current $i_{C}$ through a capacitor $C$ is,

$$
\begin{equation*}
i_{C}(t)=-C \frac{d v_{C}}{d t} \tag{659}
\end{equation*}
$$

Letting $t_{0}=0$ sec , the complex voltage $v_{C}$ across the capacitor $C$ is,

$$
\begin{gather*}
v_{C}(t)=v_{R}(t)=R i_{C}(t)=-R C \frac{d v_{C}}{d t}  \tag{660}\\
\frac{1}{v_{C}(t)} d v_{C}=-\frac{1}{R C} d t  \tag{661}\\
\int_{V_{0}}^{v_{C}(t)} \frac{1}{v_{C}(t)} d v_{C}=-\frac{1}{R C} \int_{t_{0}}^{t} d t \tag{662}
\end{gather*}
$$

$$
\begin{gather*}
\left.\ln \left(v_{C}(t)\right)\right|_{V_{0}} ^{v_{C}(t)}=-\left.\frac{1}{R C} t\right|_{t_{0}} ^{t}  \tag{663}\\
\ln \left(v_{C}(t)\right)-\ln \left(V_{0}\right)=\ln \left(\frac{v_{C}(t)}{V_{0}}\right)=-\frac{1}{R C}\left(t-t_{0}\right)  \tag{664}\\
\frac{v_{C}(t)}{V_{0}}=e^{-\frac{1}{R C}\left(t-t_{0}\right)}=e^{-\frac{1}{R C} t}  \tag{665}\\
v_{C}(t)=V_{0} e^{-\frac{1}{R C} t} \tag{666}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and/or absorbed by the resistor $R$ is,

$$
\begin{equation*}
P_{R}(t)=v_{R}(t) i_{C}(t)=\frac{v_{R}^{2}(t)}{R}=\frac{V_{0}^{2}}{R} e^{-\frac{2}{R C} t} \tag{667}
\end{equation*}
$$

Letting $t_{0}=0 \mathrm{sec}$, the energy $E_{R}$ dissipated and/or absorbed by the resistor $R$ is,

$$
\begin{gather*}
E_{R}(t)=\int_{t_{0}}^{t} P_{R} d t=\frac{V_{0}^{2}}{R} \int_{t_{0}}^{t} e^{-\frac{2}{R C} t} d t  \tag{668}\\
E_{R}(t)=\frac{V_{0}^{2}}{R}\left(\frac{-R C}{2}\right) e^{-\frac{2}{R C} t}-\frac{V_{0}^{2}}{R}\left(\frac{-R C}{2}\right) e^{-\frac{2}{R C} t_{0}}=\frac{1}{2} C V_{0}^{2}\left(e^{-\frac{2}{R C} t_{0}}-e^{-\frac{2}{R C} t}\right)  \tag{669}\\
E_{R}(t)=\frac{1}{2} C V_{0}^{2}\left(1-e^{-\frac{2}{R C} t}\right) \tag{670}
\end{gather*}
$$

Example 27. Given a time-forward voltage $V$ across capacitor $C$, a known resistor value $R$ and capacitor value $C$, compute the time-forward current flowing through the resistor, and the power and energy dissipated by the resistor.

So, given,
Direction of time $\theta=0^{\circ}$
Time 0.0 sec $\leq t \leq 1.0$ sec
Initial voltage across capacitor $V=10.0$ Volts
Capacitor $C=470 \mu F$
Resistor $R=1.0 \mathrm{k} \Omega$

The time-forward voltage $V_{0}$ across the capacitor $C$ is,

$$
\begin{equation*}
V_{0}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 0^{\circ}}=10.0 \text { Volts } \tag{671}
\end{equation*}
$$

The time-forward voltage $v_{C}$ across the capacitor $C$ is,

$$
\begin{equation*}
v_{C}(t)=V_{0} e^{-\frac{1}{R C} t}=(10.0 \text { Volts }) e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t} \tag{672}
\end{equation*}
$$

$$
\begin{align*}
& v_{C}(0.0 \mathrm{sec})=10.0 \text { Volts }  \tag{673}\\
& v_{C}(1.0 \mathrm{sec})=1.191 \text { Volts } \tag{674}
\end{align*}
$$

The instantaneous power $P_{R}$ dissipated by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{0}^{2}}{R} e^{-\frac{2}{R C} t}=\frac{(10.0 \mathrm{Volts})^{2}}{(1.0 \mathrm{k} \Omega)} e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu F)} t} t  \tag{675}\\
P_{R}(0.0 \mathrm{sec})=0.10 \mathrm{Watts}  \tag{676}\\
P_{R}(1.0 \mathrm{sec})=1.419 \times 10^{-3} \mathrm{Watts} \tag{677}
\end{gather*}
$$

The energy $E_{R}$ dissipated by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{R}(t)=\frac{C V_{0}^{2}}{2}\left(1-e^{-\frac{2}{R C} t}\right)=\frac{(470 \mu F)(10.0 \text { Volts })^{2}}{2}\left(1-e^{-\frac{2}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{678}\\
E_{R}(1.0 \mathrm{sec})=0.023 \text { Joules } \tag{679}
\end{gather*}
$$

Example 28. Given a time-advanced voltage $V$ across capacitor $C$, a known resistor value $R$ and capacitor value $C$, compute the time-advanced current flowing through the resistor, and the power and energy dissipated and absorbed by the resistor.

So, given,
Direction of time $\theta=45^{\circ}$
Time $0.0 \mathrm{sec} \leq t \leq 1.0 \mathrm{sec}$
Initial voltage across capacitor $V=10.0$ Volts
Capacitor $C=470 \mu F$
Resistor $R=1.0 \mathrm{k} \Omega$

The time-advanced voltage $V_{0}$ across the capacitor $C$ is,

$$
\begin{equation*}
V_{0}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 45^{\circ}}=7.071+7.071 j \text { Volts } \tag{680}
\end{equation*}
$$

The time-advanced voltage $v_{C}$ across the capacitor $C$ is,

$$
\begin{gather*}
v_{C}(t)=V_{0} e^{-\frac{1}{R C} t}=(7.071+7.071 j \text { Volts }) e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}  \tag{681}\\
v_{C}(0.0 \mathrm{sec})=7.071+7.071 j \text { Volts }  \tag{682}\\
v_{C}(1.0 \mathrm{sec})=0.842+0.842 j \text { Volts } \tag{683}
\end{gather*}
$$

The instantaneous power $P_{R}$ dissipated and absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{0}^{2}}{R} e^{-\frac{2}{R C} t}=\frac{(7.071+7.071 j \text { Volts })^{2}}{(1.0 \mathrm{k} \Omega)} e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu \mathrm{~F})} t}  \tag{684}\\
P_{R}(0.0 \mathrm{sec})=0.10 j \text { Watts }  \tag{685}\\
P_{R}(1.0 \mathrm{sec})=1.419 \mathrm{j} \times 10^{-3} \mathrm{Watts} \tag{686}
\end{gather*}
$$

The energy $E_{R}$ dissipated and absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{align*}
E_{R}(t)=\frac{C V_{0}^{2}}{2}\left(1-e^{-\frac{2}{R C} t}\right)= & \frac{(470 \mu F)(7.071+7.071 j \text { Volts })^{2}}{2}\left(1-e^{-\frac{2}{(1.0 k \Omega)(470 \mu F)} t}\right)  \tag{687}\\
& E_{R}(1.0 \mathrm{sec})=0.023 j \text { Joules } \tag{688}
\end{align*}
$$

Example 29. Given a time-future voltage $V$ across capacitor $C$, a known resistor value $R$ and capacitor value $C$, compute the time-future current flowing through the resistor, and the power and energy absorbed by the resistor.

So, given,
Direction of time $\theta=90^{\circ}$
Time $0.0 \mathrm{sec} \leq t \leq 1.0 \mathrm{sec}$
Initial voltage across capacitor $V=10.0$ Volts
Capacitor $C=470 \mu F$
Resistor $R=1.0 \mathrm{k} \Omega$

The time-future voltage $V_{0}$ across the capacitor $C$ is,

$$
\begin{equation*}
V_{0}=V e^{j \theta}=(10.0 \text { Volts }) e^{j 90^{\circ}}=10.0 j \text { Volts } \tag{689}
\end{equation*}
$$

The time-future voltage $v_{C}$ across the capacitor $C$ is,

$$
\begin{gather*}
v_{C}(t)=V_{0} e^{-\frac{1}{R C} t}=(10.0 j \text { Volts }) e^{-\frac{1}{(1.0 k \Omega)(470 \mu F)} t}  \tag{690}\\
v_{C}(0.0 \mathrm{sec})=10.0 j \text { Volts }  \tag{691}\\
v_{C}(1.0 \mathrm{sec})=1.191 j \text { Volts } \tag{692}
\end{gather*}
$$

The instantaneous power $P_{R}$ absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
P_{R}(t)=\frac{V_{0}^{2}}{R} e^{-\frac{2}{R C} t}=\frac{(10.0 j \text { Volts })^{2}}{(1.0 \mathrm{k} \Omega)} e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu \mathrm{~F})} t}  \tag{693}\\
P_{R}(0.0 \mathrm{sec})=-0.10 \mathrm{Watts} \tag{694}
\end{gather*}
$$

$$
\begin{equation*}
P_{R}(1.0 \mathrm{sec})=-1.419 \times 10^{-3} \mathrm{Watts} \tag{695}
\end{equation*}
$$

The energy $E_{R}$ absorbed by the resistor $R$ at $t=0.0 \mathrm{sec}$ and at $t=1.0 \mathrm{sec}$ is,

$$
\begin{gather*}
E_{R}(t)=\frac{C V_{0}^{2}}{2}\left(1-e^{-\frac{2}{R C} t}\right)=\frac{(470 \mu F)(10.0 j \text { Volts })^{2}}{2}\left(1-e^{-\frac{2}{(1.0 \mathrm{k} \Omega)(470 \mu F)} t}\right)  \tag{696}\\
E_{R}(1.0 \mathrm{sec})=-0.023 \text { Joules } \tag{697}
\end{gather*}
$$

## COMPLEX FIELD MASS FLUCTUATION TECHNOLOGIES

Since a theoretical link was established between gravity and electromagnetism, two mass fluctuation technologies are presently under investigation. Both technologies are electrical devices with the first being inductive-based, and the second being capacitive-based. Shown below is a simplified schematic diagram that highlights their operation.


FIGURE 43. Two types of complex field mass fluctuating systems.
These systems are cyclic, and alter the local gravity well. The mass of these systems is converted to excess field energy during the magnetizing/charging phase. During the demagnetizing/discharging phase, excess electrical energy is collected, and mass is restored after this phase. Then, the cycle begins again. As a consequence, clocks runs faster due to broken symmetry of mass-energy conservation in the proximity of these devices because mass is converted to NEGATIVE energy.

## AN INDICATOR OF EXCESS FREE ENERGY



FIGURE 44. Excess free energy is harvested as NEGATIVE energy.
The diagram above shows that an impulse function occurs when excess residual magnetic flux is found in the core of a coil being magnetized at the start of the next cycle, $t=0 \mathrm{sec}$. If harvested, the energy of this function manifests as NEGATIVE energy and couples to POSITIVE energy forming a complex direct current. This current consists of both real and imaginary components where the real current $i$ is time-forward and the imaginary current $j i$ is timefuture. The real current component is considered to be classic HOT CURRENT and the imaginary current component is considered to be COLD CURRENT.

Depending upon how much residual flux is available in the core, the energy in this impulse function could be quite substantial.


FIGURE 45. Excess energy found in N. Zaev’s device.

FOUR TERMINAL DEVICE


TWO TERMINAL DEVICE


$$
\begin{gathered}
E_{\text {TOT }}=E_{\text {IN }}-\left(E_{\text {OUT }}+|E|\right) \leq 0 \text { Joules } \\
C O P=\frac{E_{\text {OUT }}+|E|}{E_{\text {IN }}} \geq 1.00
\end{gathered}
$$



NOTE: ASSUME IDEAL SYSTEM

FIGURE 46. Two terminal / four terminal complex field mass fluctuating systems.
The diagram above shows a typical configuration of four terminal and two terminal complex field mass fluctuating systems. In the four terminal systems, energy from the source dipole (i.e., a battery) enters through terminals 1 and 2. Excess energy leaves through terminals 3 and 4 and charges the load dipole. In the two terminal systems, the source dipole also acts as the load dipole. Energy leaves the source dipole through terminals 1 and 2 and excess energy leaves through the same terminals at Time $t_{2}$ later.


FIGURE 47. The SmartPAK/ZPOD 1210 Workstation.


FIGURE 48. The ZPOD in operation.


FIGURE 49. The excess energy of the ZPOD.


FIGURE 50. The eBike using SmartPAK technology.


FIGURE 51. The eBike using a SmartPAK 3610-30.

## TESLA'S COMPLEX FIELD GENERATOR



FIGURE 52. Nikola Tesla's US patent 568,176.
Nikola Tesla was the first to develop the phenomenon of complex fields back in the 1880's. He devised a series of machines patented in the 1890's that greatly amplify this phenomenon, which he later called RADIANT ENERGY. As shown above, the pivoting magnetic domains created by Amperian Currents of the ferromagnetic material are ordered in the direction of field $B_{G}$ by magnetizing coil $G$. Magnetizing the high inductance coils $M$ create an opposing field $\mu_{0} H_{M}$ that acts upon the ordered domains of the material, thus canceling or partially canceling the real magnetic field created by the Amperian Currents. An imaginary magnetic field $j B_{M}$ emerges due to this cancellation and couples back into the magnetizing direct current as $i_{M} e^{j \theta}$, where $\theta>0^{\circ}$. Therefore, the magnetizing direct current becomes complex because the circulating motions of the electrons are rotating into the imaginary axis.

As shown above, before switch $F$ is closed, the capacitors $H$ are charged with a complex direct current $i_{M} e^{j \theta}$ produced by an opposing flux from coils $M$. The complex field energy is stored in capacitors $H$. At the moment of switch $F$ closure $t=0 \mathrm{sec}$, the complex direct current flows through coil $K$, rapidly discharging capacitors $H$. A very large complex electric potential $v_{L} e^{j \theta}$ is observed across the secondary coil $L$.


DENOTES HOT TIME-FORWARD ENERGY FLOW

DENOTES COLD TIME-FUTURE ENERGY FLOW
DENOTES NEUTRAL TIME-ADVANCED ENERGY FLOW
NOTE: ASSUME IDEAL SYSTEM

FIGURE 53. Tesla’s four terminal complex field generator.

THE HUTCHISON EFFECT EXPLAINED


FIGURE 54. Metal samples from John Hutchison's lab.
As shown above, John Hutchison successfully applied the Tesla complex field to metal samples with amazing results. These bulk metal samples were melted at room temperature without any application of heat. The complex fields induced cold eddy currents within the metal, which in turn, caused the metal to cold melt. As the metal softened, John inserted bits of other metals and organic material as shown. With the field turned off, the metal resolidified trapping these materials within the metal lattice structure.


FIGURE 55. John Hutchison in his lab.


FIGURE 56. Cold eddy currents being induced in a metal sample.
As shown above, cold eddy currents are induced in the metal block with the application of complex magnetic fields. A complex current flowing through the coils produces these magnetic fields. The magnetic field energy surrounding these coils is NEGATIVE, and the metal sample in the presence of this field will cold melt due to induction.


FIGURE 57. Another cold melted metal sample.

## A CONCEPT VEHICLE THAT UTILIZES GMF



FIGURE 58. A cutaway view of a capacitive-based mass fluctuating concept vehicle.
Shown above is a concept vehicle for tunneling through vast distances of space. The first step concerning this antigravitational system is the disassociation of tungsten metal with the application of a complex voltage $j V$. This metal is used as a fuel source of gravitational energy bound as mass within the element. As the metal disassociates, it radiates away this gravitational energy in the form of a temporal field within the vehicle. This causes clocks to speedup and the mass of the vehicle to decrease. The second step begins by switching off the disassociating process, thereby turning off the temporal field. Mass converted to electrical energy is amplified as field energy in the outer hull, which can be stored, radiated away, or used to power the vehicle. The vehicle seeks a new equipotential surface of gravity that corresponds to its' new mass, thus producing lift. Then, the cycle begins again.

## ENERGIZING THE GRAVITATIONAL PROPULSION UNIT (GPU)



> ACCELERATION DUE TO GRAVITY
> $\mathrm{m} / \mathrm{sec}^{2}$

FIGURE 59. Vehicle is undergoing a negative gravitational mass fluctuation above the Earth.

The diagram above shows a 1 kg vehicle undergoing a negative gravitational mass fluctuation in a given an equipotential surface of gravity reference. A typical gravity profile of the Earth shown above is based on Newton's gravity. This system decreases the relativistic mass of the vehicle such that it displaces or vectors in height to a new equipotential surface of gravity above the Earth.

Deactivating this system causes the same vehicle to naturally fall based on universal mass attraction.
Example 30. Assuming there are no other gravitational influences besides the Earth and given a vehicle of mass 1 kg positioned at an initial radius $y_{0}$ as shown in the diagram above, compute the new gravitational mass $M_{y_{1}}$ of the vehicle displaced $\Delta y$ away from the Earth.

So, given,
Mass of vehicle $M_{y_{0}}=1.0 \mathrm{~kg}$
Vehicle at initial radius $y_{0}=7.529 \times 10^{6} \mathrm{~m}$
Displacement of vehicle $\Delta y=y_{0}-y_{1}=-12.391 \times 10^{6} \mathrm{~m}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The final radius $y_{1}$ of the vehicle above the Earth is,

$$
\begin{equation*}
y_{1}=y_{0}-\Delta y=\left(7.529 \times 10^{6} \mathrm{~m}\right)-\left(-12.391 \times 10^{6} \mathrm{~m}\right)=19.920 \times 10^{6} \mathrm{~m} \tag{698}
\end{equation*}
$$

The acceleration due to gravity at altitude $y_{0}=7.529 \times 10^{6} \mathrm{~m}$ above the Earth is,

$$
\begin{equation*}
g_{y_{0}}=\frac{G M_{E}}{y_{0}{ }^{2}}=\frac{\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(7.529 \times 10^{6} \mathrm{~m}\right)^{2}}=7.038 \mathrm{~m} / \mathrm{sec}^{2} \tag{699}
\end{equation*}
$$

The acceleration due to gravity at altitude $y_{1}=19.920 \times 10^{6} \mathrm{~m}$ above the Earth is,

$$
\begin{equation*}
g_{y_{1}}=\frac{G M_{E}}{y_{1}{ }^{2}}=\frac{\left(6.67260 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)}{\left(19.920 \times 10^{6} \mathrm{~m}\right)^{2}}=1.005 \mathrm{~m} / \mathrm{sec}^{2} \tag{700}
\end{equation*}
$$

Given the exponential solution of the natural relativistic mass model, the new gravitational mass is,

$$
\begin{gather*}
M_{y_{1}}=M_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}=(1.0 \mathrm{~kg}) e^{\left(\frac{\left(1.005 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(19.920 \times 10^{6} \mathrm{~m}\right)-\left(7.038 \mathrm{~m} / \mathrm{sec}^{2}\right)\left(7.529 \times 10^{6} \mathrm{~m}\right)}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}^{2}\right.}\right)}  \tag{701}\\
M_{y_{1}}=0.9999999996333 \mathrm{~kg} \tag{702}
\end{gather*}
$$

The gravitational mass was reduced by,

$$
\begin{equation*}
M_{y_{1}}-M_{y_{0}}=(0.9999999996333 \mathrm{~kg})-(1.0 \mathrm{~kg})=-3.667 \times 10^{-10} \mathrm{~kg} \tag{703}
\end{equation*}
$$

## CLOSED-LOOP GRAVITATIONAL FLIGHT CONTROL SYSTEM



FIGURE 60. Acquire position information, then energize the GPU and vectored to new position.
The first step for a Gravitational Flight Control System, or GFCS, is to acquire vehicle height $h$ information above a known surface gravity $g_{0}$. Once this height information is acquired, the second step involves calculating the current mass fluctuation $\dot{M}_{\text {cur }}$. The third step requires the pilot to determine what the next desired position will be, so, predicted mass fluctuation $\dot{M}_{\text {pre }}$ is calculated and transmitted to a Gravitational Propulsion Unit, or GPU. The GPU is energized causing the mass of the vehicle to fluctuate, and in turn, vectors to the desired position. This system can run "closed-loop" by implementing a software algorithm called a high-speed Proportional-Integral-Derivative (PID) control loop. It acquires new height information and computes the error difference between current and desired position. The PID calculates and transmits in real-time an error value to the GPU. The error value sent to the GPU determines the rate of mass fluctuation. This rate may exceed the speed of light because the vehicle isn't traversing space by an inertial means, but by a gravitational means. Therefore, warp factor equation shown by Whitfield (1968) could be used. The warp factor $W$ equation is,

$$
\begin{gather*}
v=W^{3} c  \tag{704}\\
W=\sqrt[3]{\frac{|v|}{c}}=\sqrt[3]{\frac{\left|\frac{\Delta y}{T}\right|}{c}} \tag{705}
\end{gather*}
$$

Where, $W$ is the warp factor, $c$ is the speed of light, and $v$ is the velocity.


FIGURE 61. The warp factor vs. the speed of light.
Example 31. Assuming there are no other gravitational influences besides the Earth and given a GPU with a NEGATIVE mass fluctuation rate $\Delta M_{\text {RATE }}(t)$ operating for a period of time $T$, compute the displacement $\Delta y$ of a vehicle leaving from the surface of the Earth at warp factor $W$ with a vehicle mass of 1.0 kg .

So, given,
Initial mass of vehicle $M_{y_{0}}=1.0 \mathrm{~kg}$
Mass fluctuation rate $\Delta M_{\text {RATE }}(t)=-1.0 \times 10^{-4} \mathrm{gm} / \mathrm{sec}$
Operating time $T=1.0 \times 10^{-3} \mathrm{sec}$
Vehicle located at initial radius $Y_{0}=6.3781 \times 10^{6} \mathrm{~m}$
Speed of light $c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}$
Gravitational constant $G=6.67260 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
Mass of the Earth $M_{E}=5.9787 \times 10^{24} \mathrm{~kg}$

The new gravitational mass of the vehicle $M_{y_{1}}$ operating a GPU for a period of time $T$ is,

$$
\begin{gather*}
M_{y_{1}}=M_{y_{0}}+\left(\Delta M_{\text {RATE }}(t) \times T\right)=(1.0 \mathrm{~kg})+\left[\left(-1.0 \times 10^{-4} \mathrm{gm} / \mathrm{sec}\right)\left(1.0 \times 10^{-3} \mathrm{sec}\right)\right]  \tag{706}\\
M_{y_{1}}=0.9999999999000 \mathrm{~kg} \tag{707}
\end{gather*}
$$

Given the exponential form of the natural relativistic mass $M_{\Delta y}$ model, compute the new radius $y_{1}$ within a given gravity well $g_{y}$ is,

$$
\begin{gather*}
M_{y_{1}}=M_{y_{0}} e^{\left(\frac{g_{y_{1}} y_{1}-g_{y_{0}} y_{0}}{c^{2}}\right)}=M_{y_{0}} e^{\left(\frac{\frac{G M_{E}}{y_{1}}-\frac{G M_{E}}{y_{0}}}{c^{2}}\right)}  \tag{708}\\
\left.\frac{M_{y_{1}}}{M_{y_{0}}}=e^{\left(\frac{\frac{G M_{E}}{y_{1}}-\frac{G M_{E}}{y_{0}}}{c^{2}}\right.}\right)  \tag{709}\\
c^{2} \ln \left(\frac{M_{y_{1}}}{M_{y_{0}}}\right)=\frac{G M_{E}}{y_{1}}-\frac{G M_{E}}{y_{0}}  \tag{710}\\
y_{1}=\frac{y_{0}}{1+\frac{y_{0} c^{2}}{G M_{E}} \ln \left(\frac{M_{y_{1}}}{M_{y_{0}}}\right)}  \tag{711}\\
y_{1}=\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)}{1+\frac{\left(6.3781 \times 10^{6} \mathrm{~m}\right)\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}{\left(6.67260 \times 10^{-11} N m^{2} / \mathrm{kg}^{2}\right)\left(5.9787 \times 10^{24} \mathrm{~kg}\right)} \ln \left(\frac{0.99999999999000 \mathrm{~kg}}{1.0 \mathrm{~kg}}\right)}  \tag{712}\\
y_{1}=7.4484 \times 10^{6} \mathrm{~m}  \tag{713}\\
\Delta y=y_{0}-y_{1}=\left(6.3781 \times 10^{6} \mathrm{~m}\right)-\left(7.4484 \times 10^{6} \mathrm{~m}\right)  \tag{714}\\
\Delta y=-1.0703 \times 10^{6} \mathrm{~m} \tag{715}
\end{gather*}
$$

The warp factor $W$ is,

$$
\begin{gather*}
W=\sqrt[3]{\frac{|v|}{c}}=\sqrt[3]{\frac{\left|\frac{\Delta y}{T}\right|}{c}}=\sqrt[3]{\frac{\left|\frac{\left(-1.0703 \times 10^{6} \mathrm{~m}\right)}{\left(1.0 \times 10^{-3} \mathrm{sec}\right)}\right|}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)}}=\sqrt[3]{\frac{\left|-1.0703 \times 10^{9} \mathrm{~m} / \mathrm{sec}\right|}{\left(2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{sec}\right)}}  \tag{716}\\
W=1.5284 \tag{717}
\end{gather*}
$$

Leaving from the surface of the Earth, the vehicle is displaced over 665 miles above the Earth within 1 millisecond.

## CONCLUSION

The parameters of space and time identified as mass, volume, frequency, time, temperature, and energy are functions of gravity. Therefore, controlling the mass of a space vehicle, for example, controls gravity, but more precisely, controls its' current position within a given gravity well. The control of how fast mass fluctuates controls the speed of the vehicle through this well. Whether the vehicle implementing GMF is above the surface of the Earth or
traveling at warp factor 1.53 through deep space, the physical constants where shown in previous sections to remain invariant within the vehicle.

The vehicles’ Gravitational Propulsion Unit, or GPU, using one of the two devices that are under investigation control mass by an electrical means, thus utilizing the theoretical link between gravity and electromagnetism presented in this paper. Either device can change its’ own mass by converting it to excess electrical energy, and as a consequence, the broken symmetry of mass-energy conservation causes the gravitational energy equivalent of mass to be radiated away as a temporal field. A force is created and acts antigravitationally on the device. This changes the frequency of clocks within the vehicle relative to clocks outside of the vehicle.

The vehicle will require a Gravitational Flight Control System, or GFCS, capable of controlling the GPU. This control system acts as an interface between man and the GPU. The development of this system can be directly implemented from the mathematical formulations in this paper. An additional requirement is the development of real-time navigation software programs that maps the gravity of the entire Sol System, and includes the nearby stars.

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